Using Maxwell's Counting by Tens Strategy to Solve a Groups-of-Ten Word Problem

This story is a part of the series:

What's Next? Stories of Teachers Engaging in Collaborative Inquiry Focused on Using Student Thinking to Inform Instructional Decisions

What's Next?

Stories of teachers engaging in collaborative inquiry focused on using student thinking to inform instructional decisions

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Introduction

This lesson illustrates how multiplication and division word problems involving groups of ten can be used to assess and develop first and second graders' understanding of place value concepts in base-ten numbers. First, two word problems involving groups of ten were posed to a class of second-grade students in late September as a way to assess the degree to which the students were already using knowledge of the base-ten number system in their solutions. On the basis of what was learned, a lesson for the second-grade class was created with the goal of advancing students' understanding of place value concepts. The lesson focused on discussion of students' varied ways of figuring out how many $10 pizzas can be purchased with $140.

Relevant Florida Mathematics Standards

MAFS.2.NBT.1.1 Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases:

a. 100 can be thought of as a bundle of ten tens – called a "hundred."

b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).

MAFS.3.OA.1.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

Background Information

Consider reading chapters four and six from *Children's Mathematics: Cognitively Guided Instruction* (Carpenter et al., 2015). Chapter four provides background on word-problem types that represent basic multiplication and division and the strategies early-grades learners typically use to solve them. Chapter six explores how multiplication and division word problems involving grouping by tens can be used to help students to develop their understanding of the base-ten number system.

Carpenter, T. P., Fennema, E., Franke, M. L., Levi, L., & Empson, S. B. (2015). *Children's Mathematics: Cognitively Guided Instruction*. Portsmouth, NH: Heinemann.

Analyzing Student Thinking

The following problems involving groups of ten were posed to a class of second-grade students in late September:

A. *I have 8 boxes with 10 pencils in each box. How many pencils do I have?* (Multiplication)

B. *The second graders at Orange Grove School raised $67 to buy books for the children's hospital. If each book costs $10, how many books can they buy?* (Measurement division)

These problems were posed to students without any instructions about how to solve them as a way to get an unbiased assessment of their thinking processes. Students were encouraged to pay close attention to the problem situations and to generate solutions using tools (e.g., manipulatives, fingers, writing materials) or mental strategies of their own choosing.

As the students worked, the teacher observed each student's ways of solving the two problems and classified their strategies into the following categories[1].

The student who uses a *counting by ones* strategy represents all the quantities in the problem (grouping ten in each group) and counts by

1 The descriptions of strategies presented in this section are the current descriptions used by our team. We consider them to be fluid and open for discussion as our understanding of these ideas continually evolve. For a more detailed introduction to these terms, consider reading Carpenter et al. (2015).

Problem	Strategies		
	Counting by ones	*Counting by tens*	*Direct place value*
Multiplication *I have 8 boxes with 10 pencils in each box. How many pencils do I have?*	Draws eight boxes with ten dots in each box. Counts all of the dots by ones.	Counts by tens keeping track on fingers, "10, 20, 30, 40, 50, 60, 70, 80." *or* Uses eight cubes to represent the boxes, then puts a ten rod underneath each cube (to represent the ten pencils in each box). Counts the ten rods by tens to 80.	Quickly identifies the answer as 80, without apparent counting. Justifies answer by stating, "eight tens make 80."
Measurement division *The second graders at Orange Grove School raised $67 to buy books for the children's hospital. If each book costs $10, how many books can they buy?*	Counts out 67 cubes by ones. Groups the cubes in sets of ten until no more sets of ten can be made. Determines the answer by counting the sets of ten, "1, 2, 3, 4, 5, 6."	Represents 67 with base-ten blocks by first pulling ten rods, one at a time saying, "10, 20, 30, 40, 50, 60" and then pulling seven unit cubes. Determines answer by counting the ten rods, "1, 2, 3, 4, 5, 6." *or* Counts to 60 by tens, "10, 20, 30, 40, 50, 60," extending a finger for each count. Counts or observes that six fingers are extended to determine the answer.	Says, "Six. There are six ten dollars in 67 dollars."

Figure 1. Three levels of sophistication in the strategies students typically use to successfully solve grouping-by-tens problems.

ones to determine the solution.

The student who uses a *counting by tens* strategy represents all the quantities and counts by tens and ones, keeping track with manipulatives or drawings (including fingers).

The student who uses a *direct place value* strategy knows how many tens and ones are in the given number and provides the answer without counting or creating a physical representation.

Figure 1 illustrates these strategies in relation to the two problems that were posed to the second graders.

A chart summarizing the teacher's classification of her students' strategies is presented in Figure 2. Although Tiffany and Cora counted by ones to solve the multiplication problem, they both used a more sophisticated strategy using ten as a unit for the measurement division problem. Although most of the students with viable strategies used their fingers or base-ten blocks to support counting by tens on at least one of the problems, Kyle and Brianna were able to solve both problems confidently using their knowledge of place value (with a *direct place value* strategy) rather than counting. Xavier, Mia, Paul, and Tristan struggled to interpret one or both problems, applying strategies that did not reflect understanding of the problem situations.

Developing the Learning Goal

From observation of how the second-grade class solved these problems, the teacher determined that at least 75% of her students were exhibiting some degree of base-ten thinking within 100, and she hypothesized that the class was ready to explore similar groups-of-ten problems involving quantities greater than 100. The teacher observed that many of her students were counting by tens at some point in the process of solving these two problems. Based on this information about her class, the teacher established the following goal for the next lesson:

> *Deepen students' understanding of base-ten concepts, including the idea that ten tens is the same amount as one hundred, through discussion of student-generated solutions to a groups-of-ten task involving a three-digit number.*

Planning for the Lesson

While they were solving the two problems, the teacher noticed that more students struggled to interpret the context of the measurement-division word problem than the multiplication problem. She also thought that the context of the multiplication problem (i.e., pencils) was more concrete and familiar to her students than the context of the measurement-division problems (i.e., dollars), and that may have affected the relative difficulty of the problems.

In an effort to stimulate student thinking about the fact that ten tens is the same amount as 100 and to provide students with another measurement-division experience, the teacher decided to design a lesson focused on the following measurement-division problem involving sets of ten with a decade number as the dividend:

> *I have $140 to spend for a pizza party. If every pizza costs $10, how many pizzas can I buy?*

The teacher felt that this provided a next-step challenge for all of the students who had used the direct place value strategy on the interview problem with numbers within 100. She selected 140 as the dividend (i.e., the number to be divided by another number), because she thought the magnitude would provide some challenge to students, partly because it would require counting more than 100. She limited the complexity of the problem by making the dividend a multiple of ten.

Even with these changes to the problem, the teacher acknowledged that budgeting for a party may still be a difficult scenario for her students to comprehend. In an effort to increase comprehension of the situation, she planned to introduce the problem slowly to the students. She also decided to introduce it much more like she was telling a story than posing a typical three-sentence word problem.

Based on the ways students solved the two initial problems, the teacher anticipated her students would use the following types of strategies to solve the new problem.

	Strategies			
Problem	*Counting by ones*	*Counting by tens*	*Direct place value*	*Other*
Multiplication *I have 8 boxes with 10 pencils in each box. How many pencils do I have?*	Tristan Tiffany Cora	Logan Paul Kiara Gina Parker Maxwell Caden Tariq Lilly Cora	Kyle Brianna	Xavier Mia
Measurement Division *The second graders at Maplewood School raised $67 to buy books for the children's hospital. If each book costs $10, how many books can they buy?*		Logan Kiara Gina Maxwell Caden Cora	Lilly Tariq Parker Tiffany Kyle Brianna	Paul Xavier Tristen Mia

Figure 2. Strategies used by these second-grade students.

Counting by ones

A. Some students might attempt create a set of 140 objects and count them by ones.

Counting by tens

B. Some students might put ten rods in the work space, counting 10, 20, 30, 40 . . . 140. They would then determine the answer by counting the ten rods (each rod simultaneously representing a pizza and $10).

C. Some students might represent 140 with one hundred flat and four ten rods. They might know that the square is ten pizzas and then and then use a count on from ten strategy with each of the ten rods, 11, 12, 13, 14. Students might also start with this strategy (building 140 with one hundred flat and four tens rods) but then not know what to do to find the answer.

D. Some students might count by ten to 140 without manipulatives, keeping track of their counts on fingers, with tally marks, or with some other notation.

Direct place value

E. Some students might use direct place value to recognize that a 1 appears in the hundreds place, and this is the same amount as ten tens (or ten pizzas). The 4 in the tens place corresponds to four more pizzas (10 + 4 = 14).

F. Some students might see 140 and simply know that it is the same amount as 14 tens.

Nonvalid strategies

G. Some students might say that the answer is four, because there is a four in the tens place. This might be the response of a student attempting a direct place value strategy without thinking about the reasonableness of the solution.

H. Some students might add ten to 140 or subtract ten from it, because he or she is doing something with the numbers in the problem without making sense of the problem situation.

To work toward the established lesson goal, the teacher set out to implement the pizza party problem with these specific teaching strategies in mind:

- To assist students who are struggling with interpreting the problem situation with considering how to make sense of problem situations involving groups of ten through modeling with manipulatives.
- To encourage students who are sometimes *counting by ones* to explain their own or other students' *counting by tens* strategies.
- To encourage students who are using *counting by tens* strategies to explain the relations between their strategies and their classmates' *direct place value* strategies.
- To make explicit how each solution is related to the problem context (e.g., Where in the solution are the 140, the tens, the pizzas?)

Lesson Plan

The lesson plan that follows was developed in attempt to work toward the following goal:

> *Deepen students' understanding of base-ten concepts, including the idea that ten tens is the same amount as one hundred, through discussion of student-generated solutions to a groups-of-ten task involving a three-digit number.*

1. Share with students that you would like their help figuring something out. Use story telling to establish a verbal scenario that makes the problem personal and motivating to students. Say: *A bunch of teachers are going to be staying at school into the evening tonight. Mr. P (the principal) has given me $140 to buy pizza for the teachers. He wants me to order the pizzas from a restaurant where the pizzas cost $10 each. I hope you can help me figure out how many pizzas I can buy with the $140.*

2. Post a written version of the problem and read it aloud: *I have $140 to spend for a pizza party. If every pizza costs $10, how many pizzas can I buy?*

3. Establish expectations and logistics for work time:

   a. Every student should initially work independently to solve the problem in a way that makes sense to him or her.

   b. Students can choose to use any manipulatives in the bins provided for each table (e.g., base-ten blocks, snap cubes), if desired. Manipulatives should be used to solve the math problem and not for building or playing. Students may also solve without manipulatives, using pictures, numbers, or mental strategies.

   c. Students will have approximately ten minutes of work time, and then the class will discuss the different ideas students have for solving the problem.

   d. Students should record their mathematical process with pictures, numbers, words, or some combination in writing or be ready to show how they used manipulatives to solve the problem.

4. Have students reread the problem and begin working.

5. As students begin working, approach those who have struggled to interpret the initial assessment problems and ask them to retell the problem in their own words. Encourage rereading and retelling until the student appears to understand the details of the problem, including the question.

6. Circulate and observe students' ways of approaching the problem. Ask questions about the details of their mathematical strategies, make personalized task modifications, and prompt student-to-student discussion among early finishers:

   a. If a student appears to have used a *direct place value* strategy (i.e., he or she knows the answer quickly and without apparent counting), ask the student to explain his or her thinking verbally and then in writing. Assess the degree to which the student can articulate a justification for the answer. As time allows:

      i. Consider pairing the student using a *direct place value* strategy with a student who has used a manipulatives-based *counting by tens* strategy to share and compare solutions.

      ii. Consider having the student who is using a *direct place value* strategy generate a second solution strategy.

   b. If a student uses a *counting by tens* strategy with base-ten blocks (or base-ten block quick pictures), note whether the student uses 14 ten rods or uses one hundred flat and 4 ten rods. If the former, note

These problems were posed to students without any instructions about how to solve them as a way to get an unbiased assessment of their thinking processes. Students were encouraged to pay close attention to the problem situations and to generate solutions using tools (e.g., manipulatives, fingers, writing materials) or mental strategies of their own choosing.

In accordance with her goal of stimulating thinking about base-ten ideas, the teacher determined that she wanted to focus class discussion on the counting by tens strategies of Paul, Gina, and Maxwell. She would decide whether to ask Caden to explain the direct place value strategy he used based on what she observed in the resulting class discussion.

whether the student knows that the hundred flat is ten tens or needs to count the tens on the square. As time allows:

i. Consider asking: How are you thinking about this problem? Where in your model is the $140? Where is one pizza? How did you determine that I can buy ___ pizzas?

ii. Consider pairing this student to share and compare with a student using a different *counting by tens* strategy, *counting by ones*, or *direct place value*.

c. If a student uses a *counting by tens* strategy without manipulatives or pictures, prompt the student to describe the details of the strategy and support the student with notating the strategy, if necessary. As time allows:

i. Consider asking: How are you thinking about this problem? How did you know when to stop counting by tens? How did you decide how many pizzas I can order? With your strategy, what represented 1 pizza?

ii. Consider pairing this student to share and compare with a student using a *counting by tens* strategy with manipulatives.

d. If a student uses a *counting by ones* strategy, monitor the student's level of frustration. If frustration is high, consider modifying the $140 in the problem to $90 or $60.

e. If a student has used an *non-valid* strategy (i.e., misinterpreted the problem), use one or both of the following strategies, as time allows:

i. Consider posing questions to the student to explain the strategy he/she is using and gain insight into the basis of the misinterpretation. For example: What did you do first? What in the problem made you decide to do that?

ii. Consider pairing the student to share and compare with a student who has used a clear and concrete *counting by ones* or *counting by tens* strategy.

7. Identify three to four students whose solutions you want to involve as the targets of class discussion. Ask each of these students whether you may share his or her work in the class discussion. If you are going to ask the student to explain his/her work, be cognizant of the student's comfort level with sharing. Look specifically for the following solutions:

a. A student who modeled 140 using 14 tens and counting by tens.

b. A student who modeled 140 using one hundred flat and four tens and who knew that the hundred flat was ten tens without counting.

c. A student who used a *direct place value* strategy and can articulate *just knowing* that ten tens is one hundred, so that is ten pizzas.

8. When you observe that the students have had enough time to generate personal solutions, convene a class discussion in which you first call on individual students to review the important details of the problem. Consider calling on students who have previously struggled to interpret problems.

9. Explain that you have picked out a few students' solutions to discuss as a class. State explicitly that, as the class looks at each solution, every student's job is to try to understand what the student was thinking and to ask questions when they are not sure.

10. Guide students to examine, one at a time, the three to four solution strategies identified during work time. Sequence the sharing of strategies from less to more sophisticated. This organized sequence may provide the students in the class with opportunities to make sense of increasingly abstract strategies and draw connections among them. In accordance with the established goal:

    a. Provide opportunities for the students to explain their understanding of the solution strategies of others in relation to the problem context.

        i. Consider sometimes presenting student work visually to the whole class and having the class make conjectures about what the student did. Then the student whose work is being shared can verify or refute the conjectures.

        ii. Ask questions to help students notice important aspects of each solution (e.g., How does this solution show the $140? How did *Student A* know to stop counting tens? Where are the pizzas represented in this solution?)

    b. Take advantage of opportunities to highlight the relation between ten tens and 100 in students' solutions.

        i. When a strategy uses 100, prompt the class to consider why the student did that. How many tens are in 100?

        ii. When a strategy does not explicitly use 100, consider having students identify 100 in the solution.

        iii. Compare and contrast solutions that use 100 with similar strategies that do not.

    c. Deliberately call on particular students to provide explanations or answer questions.

        i. Invite students who have struggled to interpret the problem to answer questions that probe the relation between the problem context and students' solutions.

        ii. Invite students who use *counting by ones* to explain the most concrete *counting by tens* strategy.

        iii. Invite students who have used *counting by tens* strategies that do not highlight 100 to explain such strategies that do.

        iv. Invite students who have used *direct place value* to explain their classmate's physical and pictorial models.

Reflection

What happened in the classroom

After the teacher introduced the pizza party story problem, most students started collecting manipulatives or drawing, but a few sat pensively. These students might have been working the problem mentally, or they might have been stuck.

One student (Caden) blurted the answer (14) aloud almost immediately. The teacher approached Caden discreetly and asked him to explain his thinking, first to her and then in writing.

Next, she visited the desks of two students who had not started yet. Both of them had experienced difficulty interpreting the previously posed groups-of-ten problems, so the teacher focused her efforts on helping these students to review and explain the context of the pizza-party problem.

She then circulated around the room, observing students' work and asking a few students questions about their strategies (e.g., How are you thinking about this problem? Why did you…?) The teacher noted that most students used strategies similar to those they used on the previous problems. Two students approached the pizza-party problem with a *counting by ones* strategy, and two appeared to use a *direct place value* strategy. The majority, however, used a *counting by tens* strategy, some representing 140 as 14 tens (with either base-ten blocks or pictures) and others representing 140 with a physical or pictorial representation of 100 and 4 tens.

After all students had solutions (approximately ten minutes), the teacher initiated class discussion emphasizing her expectation that the students work hard to make sense of each others' work and mathematical thinking. In accordance with her goal of stimulating thinking about base-ten ideas, the teacher determined that she wanted to focus class discussion on the *counting by tens* strategies of Paul, Gina, and Maxwell. She would decide whether to ask Caden to explain the *direct place value* strategy he used based on what she observed in the resulting class discussion.

After a brief review of the pizza-party problem, the teacher helped Paul display his work (See Figure 3) on the document camera and challenged the class to make conjectures about what Paul did and how he decided the answer was 14.

In discussion of Paul's strategy, the teacher intentionally directed questions to students who had used a *counting by ones* strategy, in an effort to support those students in making sense of how 140 was composed of tens. First, the class estab-

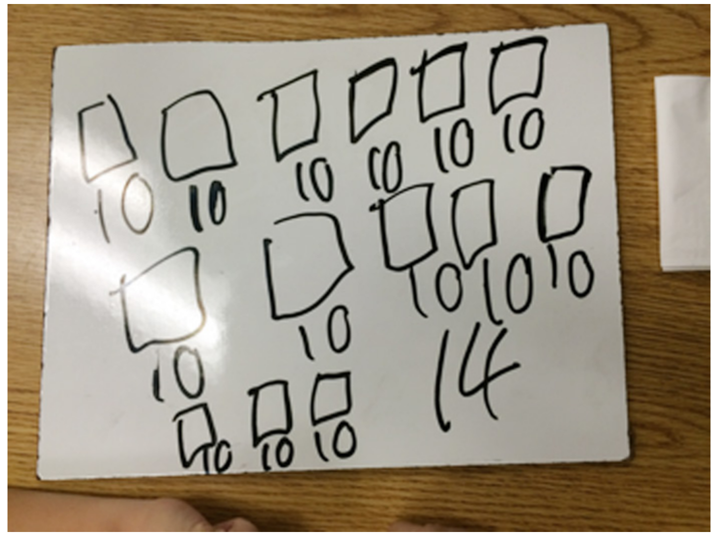

*Figure 3. Paul's **counting by tens** strategy for the pizza-party problem.*

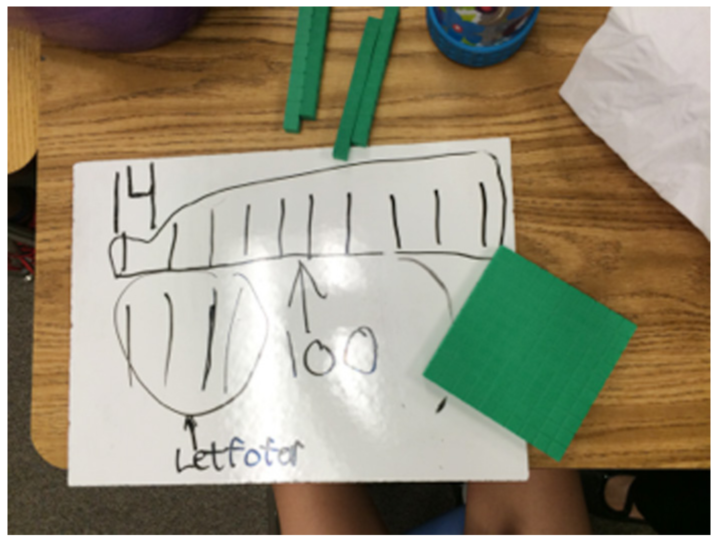

*Figure 4. Gina's **counting by tens** strategy for the pizza-party problem.*

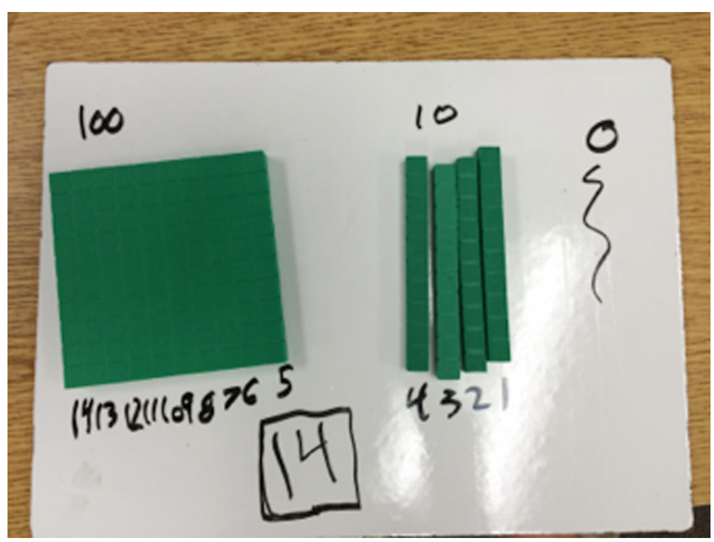

*Figure 5. Maxwell's **counting by tens** strategy for the pizza-party problem.*

lished that Paul had made a square to represent each pizza and that he had written the tens underneath the squares to indicate that each pizza was $10. The teacher then asked the class to explain how Paul knew when to stop drawing pizzas. The class quickly reasoned that Paul must have been counting by tens; but extended discussion and class reenactment of the skip-counting-by-tens sequence were needed for the class as whole to make sense of how Paul used skip-counting by tens in coordination with his drawing to determine when to stop counting (at 140). For many of the students, skip-counting across the benchmark 100 was difficult, so discussion of Paul's strategy also offered a nice opportunity to extend skip-counting by tens beyond 100 in a meaningful context.

Gina's strategy (See Figure 4) became the object of focus for the next segment of class discussion.

After helping students unpack the details of how Gina generated her drawing of 14 base-ten rods using skip-counting in a way that was similar to the way Paul counted, the teacher attempted to direct the students' attention to the ring in Gina's picture that was labeled 100. She removed the solution from the document camera and challenged students to think about how many base-ten rods were in the ring. After she "flashed" the solution on the document camera, the class conjectured and verified that the ring encompassed ten base-ten rods and that ten tens was the same amount as 100. The teacher then had the class consider how they might label 100 in Paul's picture. Toggling back and forth between Gina and Paul's representations in the discussion, the idea that 140 is composed of 100 and 40 seemed to become more clearly established.

Next, the class examined Maxwell's representation of 140 with 1 hundred flat and 4 ten rods (See Figure 5). In contrast to Paul and Gina, who both created 140 by counting by tens starting at ten, Maxwell used 100 as a starting place and then counted on by tens (110, 120, 130, 140).

The teacher used discussion of Maxwell's strategy to stimulate student thinking about how tens are embedded in hundreds, with particular focus on finding the tens (or pizzas) in the large green square. After allowing students time to study Maxwell's solution, the teacher asked, "Why did Maxwell write the numbers underneath?" Through the conversation that followed, the class determined that the numbers under the rods or columns in the square were counting the pizzas and that each rod or column was both a pizza and $10.

Finally, the teacher asked Caden to share his *direct place value* strategy to expose students who were already using *direct modeling with tens* to this more sophisticated strategy. Caden shared that he thought about the fact that 100 has ten tens and that 40 has four tens, and 10 + 4 = 14. As Caden explained, the teacher wrote the following statements on the board:

140
100 has ten tens in it.
40 has four tens in it.
10 + 4 = 14

Although the teacher thought this strategy might be too abstract for some students in the class to understand fully, a few students asked clarifying questions about the strategy, and one shared his on-the-spot recognition that the number 140 contains 14 and is composed of 14 tens.

Planning for Future Lessons

In reflecting on the lesson, the teacher observed that all students were able to generate a productive and correct solution strategy—two used a *counting by ones* strategy, two used *direct place value*, and the rest used *counting by tens*. Her minimal intervention at the beginning of work time to help two students to understand the problem context seemed successful. She did not take advantage of the strategy of having partners share and compare their solution strategies during work time as she had originally intended; that will be a strategy for a future lesson. During the class discussion, the teacher was deliberately choosing students who had used counting by ones strategies to explain how their classmates took advantage of strategies involving counting by tens. She did so in hopes of moving students toward using strategies that capitalize on the base-ten structure of our number system.

On the basis of the ways students solved and discussed the solutions of others for the pizza-party problem, the teacher determined that her next step would be to implement additional multiplication and measurement division problems involving sets of ten in which the product or dividend was a multiple of ten between 100 and 200. In posing additional problems of these kinds, she hopes to see whether students who used counting by ones on this problem will move toward more efficient counting by tens strategies. She is also interested to see whether more students will start to integrate their emerging knowledge of the relation between ten tens and one hundred into their solution strategies.

What's Next?

Stories of teachers engaging in collaborative inquiry focused on using student thinking to inform instructional decisions

What's Next? is a collection of stories documenting professional development experiences shared by elementary teachers working collaboratively to study the complex process of teaching and learning mathematics. Each story in the collection describes practicing teachers studying the thinking processes of real students and using what they learn about those students to make decisions and try to help advance those students' understanding on that day.

The teachers in each story start by learning about how individual students are solving a set of mathematics problems. They use this freshly gathered knowledge of student thinking to develop near-term learning goals for students and a lesson plan tailored to specific students on that specific day. One of the teachers implements the planned lesson while the other teachers observe in real time. The teachers then gather to discuss and reflect on their observations and insights.

In these lessons, the practice of teaching is slowed way down. The stories tell of teachers who are studying student thinking and using that information to plan and implement instructional decisions at a pace that is much slower than it occurs in daily practice. The stories in this collection also depict many aspects in common with formative assessment and lesson study, both of which are a process and not an outcome.

The stories depict real situations that occurred in real time and include both successes and shortcomings. We hope that the stories may be studied and discussed by interested educators so that the lessons and ideas experiences of these teachers and instructional coaches may contribute to additional learning and sharing among other interested teachers.

Learn more about these and other stories at **http://www.teachingisproblemsolving.org/**