



# Teachers Analyze Student Thinking and Write Detailed Learning Goals for Each Student

This story is a part of the series:

***What's Next? Stories of Teachers Engaging in Collaborative Inquiry Focused on  
Using Student Thinking to Inform Instructional Decisions***

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# What's Next?

Stories of teachers engaging in collaborative inquiry focused on using student thinking to inform instructional decisions

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## Introduction

This lesson centers around students' explanations of their strategies to solve a word problem with multidigit numbers. The lesson followed a series of one-on-one interviews with first-grade students. After the information gained during those interviews was analyzed, the lesson focused on helping students to use more sophisticated strategies to perform multidigit addition and to learn to express their mathematical ideas with drawings, manipulatives, and abstract-symbolic notation. The primary approach involves asking children to explain their thinking, listen to one another, and strive to make sense of one another's ideas. The teacher orchestrated the discussion and provided guidance to students to help them learn to express their mathematical ideas using pictures, manipulatives, and equations.

## Relevant Florida Mathematics Standards

**MAFS.1.NBT.2.2** Understand that the two digits of a two-digit number represent amounts of tens and ones.

- a. 10 can be thought of as a bundle of ten ones—called a “ten.”
- b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.
- c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).
- d. Decompose two-digit numbers in multiple ways (e.g., 64 can be decomposed into 6 tens and 4 ones or into 5 tens and 14 ones).

**MAFS.1.NBT.3.4** Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Under-

stand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.

## Background Information

This investigation was strongly informed by chapters two, three, six, and seven in *Children's Mathematics: Cognitively Guided Instruction*. These chapters provide background on addition and subtraction word problems as well as strategies that students use to solve these types of problems. In addition, chapter six provides some background information on base-ten number concepts. Chapter seven provides additional information about children's strategies when they solve problems involving multidigit numbers.

Carpenter, T. P., Fennema, E., Franke, M. L., Levi, L., & Empson, S. B. (2015). *Children's Mathematics: Cognitively Guided Instruction* (2nd ed.). Portsmouth, NH: Heinemann.

## Analyzing Student Thinking

In one-on-one interviews, a group of teachers posed the following four problems to each student in a class of first-grade students. Each teacher was assigned to one student in the class to interview individually with the goal of obtaining a detailed understanding of each child's mathematical understanding related to addition, subtraction, and place value on that day. The students had access to linking cubes, base-ten blocks, pencils, and paper. Students were encouraged to solve the problems in a way that made sense to them, and they were invited to use any of the available tools that they might need. The interviewing teacher read each problem to the student, and the reading of the problem could be repeated as needed. After providing an answer, the student was asked to explain how he or she solved each problem.

- A. Pete had 20 rocks. Juan gave him 24 more rocks. How many rocks does Pete have now?
- B. Tylesha had 32 books. Her grandma gave her 25 more books. How many books does Tylesha have now?

- C. Mr. Jones had 40 cupcakes. He gave 20 cupcakes to the students in his class. How many cupcakes does Mr. Jones have now?
- D. Maria had 35 jellybeans. Her dad gave her 27 more jellybeans. How many jellybeans does Maria have now?

The following provides a quick summary of the working definitions of the student strategy categories that were used to sort the students' work for the Tylesha's-books problem.<sup>1</sup>

### *Multidigit Computation Strategies*

A student who uses a *direct modeling with ones* strategy represents each multidigit number in the problem as a set of ones using manipulable objects or pictures to represent the story in the problem and then counts the objects or pictures to determine the answer. In the Tylesha's-books problem, a student using a *direct modeling with ones* strategy might create a set of 32 individual objects, create another set of 25 individual objects, put the two sets of objects together, and count the objects in the combined set by ones.

A student who uses a *counting by ones* strategy does so without physically representing every quantity in the problem. Fingers, objects, or tally marks are often used to keep track of the number of counts. In the Tylesha's-books problem, a student using a *counting by ones* strategy might say "32" and proceed to count forward by ones 25 times while keeping track of the count with fingers.

A student who uses a *direct modeling with tens and ones* strategy represents each multidigit number in the problem using manipulable objects or pictures that reflect the base-ten structure of the number system. The student then counts the objects or pictures by tens and ones to determine the answer. In the Tylesha's-books problem, a student using a *direct modeling with tens and ones*

strategy might use base-ten blocks to model both 32 and 25 by representing 32 with three ten rods and two unit cubes then model 25 with two ten rods and five unit cubes. The student might then count the number of objects, either by ones or by tens and ones. For example, the student might say, "10, 20, 30, 40, 50, 51, 52, 53, 54, 55, 56, 57." (As one of many variations within this type of strategy, the student could also count the ten rods by ones and consider five ten rods to be 50.) If the student counts from 1 to 57 by ones, the student's strategy is still considered *direct modeling with tens and ones*, because the student created a representation involving groups of tens and ones.

A student who uses a *combining like units* strategy operates on the numerals representing groups of tens and ones separately, then combines the partial sums or differences to get a final result. In the Tylesha's-books problem, the student might think about adding three tens and two tens to make 50, and adding two and five to make seven. The student might then say, "50 + 7 is 57."

The student who uses an *incrementing* strategy determines the answer by increasing or decreasing partial sums or differences. In the Tylesha's-books problem, a student using an *incrementing* strategy might say, "30 + 20 is 50, 50 + 5 is 55, and 55 + 2 is 57."

### *Strategies used by students in this class*

After interviewing the students in the class and reflecting on the strategies the students used to solve the problems, the teachers created a chart with categories of students' strategies (Figure 1).

As shown in Figure 1, despite teachers' anticipation that they would, no students used a *counting by ones* strategy or an *incrementing* strategy.

Christopher, Mikhail, and Elizabeth are all listed in the *direct modeling with ones* category because, even though each of them used tens and ones in their concrete representations, when they counted their sets, they counted each "unit" on the ten rod by ones. The teachers decided that these three students did not have a firm understanding of the ten rod as a unit of ten and therefore be-

<sup>1</sup> The descriptions of strategies presented here are the current descriptions used by our team, and we consider them to be fluid, as our understanding of these ideas continues to evolve. For a more detailed discussion of these terms, consider reading Carpenter et al. (2015).

<i>Direct modeling with ones</i>	<i>Direct modeling with tens and ones</i>	<i>Combining like units</i>	<i>Nonvalid strategy</i>
Amiah	Alyson	Kenneth	Emily
Makayla	Wendy	Jude	Nicholas
Christopher*	Mason		Shelby
Elizabeth*	Annaliz		Jackline
Mikhail*			Alexis
			Landon

\*Used groups of ten in representations.

Figure 1. Teachers' classification of students' strategies for the Tylesha's-books problem

longed in the *direct modeling with ones* category. Several students used a *direct modeling with tens and ones* strategy. Alyson and Wendy both represented both numbers with groups of ten and ones. When they counted their sets, they counted the first set by tens and ones and then counted the second set by ones. The teachers decided this is an early example of a strategy that falls within the *direct modeling with tens and ones* category.

### Learning Goals

In discussing the appropriate learning goals for these students, the teachers recognized the class included many different areas of need and concluded that it would be useful to consider learning goals that reflected the needs of each individual student. On the basis of how the students solved the Tylesha's-books problem, the teachers created the following list of student-specific learning goals for the new lesson.

*Jackline will represent the quantities accurately.*

*Alexis will understand the difference between a ten and a one.*

*Landon will use one to one correspondence when counting sets of objects.*

*Students who counted incorrectly as a result of poor organization of their concrete representations of the problem will recognize ways to separate the sets so that counting the objects is easier.*

*Students who used a direct modeling by ones strategy will recognize a group of 10 and that*

*32 can be represented with three tens and two ones.*

*The three students (Christopher, Elizabeth, and Mikhail) who used a direct modeling by tens strategy but counted the sets by ones will learn and be able to explain that 32 is made of three tens and two ones.*

*The two students (Shelby and Wendy) who used a direct modeling by tens strategy and counted the first set by tens and ones, but then counted the second set by ones will learn and be able to explain that they can group the tens together and then count the full set by tens and then by ones.*

*Students who used a combining like units strategy will learn to express their mathematical ideas in various representations and apply this strategy in a variety of contexts and with more challenging number combinations.*

The teachers decided to design a lesson that offered opportunities to create connections between the *direct modeling with ones* and the *direct modeling with tens* strategies. They also thought that Alyson and Wendy would benefit from practicing grouping tens together and ones together in their representations to help them count more efficiently. Finally, those students who struggled with the counting sequence when *counting by ones* could benefit from hearing and helping other students count aloud.

Summarizing the discussion, the two overarching learning goals for the lesson were as follows.

*Students will develop their abilities to use and*

*interpret direct-modeling strategies involving groups of tens and ones.*

*Students will learn to express their ideas about place value and addition of multidigit numbers using pictorial representations and abstract-symbolic notation and will understand the conceptual connections among the various representations.*

## Planning for the Lesson

Through extensive discussion and analysis of the learning goals, the teachers created the following word problem to use in the impending lesson.

*Amiah has 34 sea shells. Her friend Alexis gave her 23 more sea shells. How many sea shells does Amiah have now?*

### *Rationale for the Problem Selected*

The teachers chose to use a *join result unknown* problem. This is one of the easiest types of addition word problems, and the class had already demonstrated that they could successfully solve such problems. They decided to use this simple problem type so that students could focus on the numbers and counting strategies rather than getting bogged down with the more cognitively challenging situations in *compare*, *change unknown*, or *start unknown* situations. The numbers 34 and 23 were chosen, because they lend themselves to counting by tens and ones without having more than ten ones.

The teachers then analyzed the classification chart (Figure 1) and identified those students who would be ideal for sharing their strategies from the Tylesha's-books problem with the class and worked to develop the best sequence in which those students should share their strategies. They decided that, as students shared their strategies, an effort would be made to connect the ideas presented in the various strategies across the progression. Students who struggled to count would be asked to listen to others counting the objects in their models and would be asked to count verbally along with their peers.

*The teachers agreed that most students in this class were ready to solve problems involving numbers that would require them to regroup the ones to make another ten, such as a join result unknown problem with the numbers 28 and 56.*

The teachers determined that the students should share in the following order:

1. Makayla—This student was selected to share first because she used a *direct modeling with ones* strategy. The teachers thought Landon, Alexis, and Jackline might benefit from hearing a student count by ones.
2. Wendy would be asked to share next, because she used a directing modeling with tens strategy, and she counted the first set by tens and ones and the second set by ones. Either Elizabeth or Mikhail would be asked to explain how Wendy's strategy differed from theirs.
3. If time permitted, students who used a *combining like units* strategy would be asked to share their strategies.

As students shared their strategies, the teacher would listen for opportunities to make connections between different students' strategies. In addition, the teacher would take note of those students who were making their own connections to ways their strategy was similar to or different from the shared strategies.

While students solved the new problem, the teacher would identify the students who would be asked to share their thinking with the rest of the class at the end of the lesson. The teacher would look specifically for variations on the theme of the *direct modeling with tens* strategy. The teacher would also make note of students who use a strategy different from the one they used to solve the Tylesha's books problem.

## Lesson Plan

A reminder—on the basis of how the students solved the Tylesha's books problem, teachers determined they would use the following two overarching learning goals to guide decisions during the classroom lesson:

*Students will develop their abilities to use and interpret direct-modeling strategies involving groups of tens and ones.*

*Students will learn to express their ideas about place value and addition of multidigit numbers using pictorial representations and abstract-symbolic notation and will understand the conceptual connections among the various representations.*

### Lesson narrative

The teacher said, "I am going to give you back the problem that you solved earlier. I want you to think about this problem and how you solved it." As she said this, the teacher distributed the Tylesha's-books problem from the interview back to students.

The teacher said, "You solved the following problem this morning. Tylesha had 32 books. Her grandma gave her 25 more books. How many books does Tylesha have now? I want you to think about how you solved that problem. I am going to ask some of you to come share your strategies with the rest of the class."

The teacher asked, "Who can tell me what the problem was about?" The teacher invited several students to explain what the problem was about.

After a brief discussion about the context of the problem, the teacher said, "Makayla, can you come up and show us how you solved the problem?"

Makayla put her paper on the document camera to display it to the class, as shown in Figure 2.

The teacher asked, "Makayla, can you tell us what all these (the boxes) are on your paper?"

Makayla said, "Half are 32 books and half are 25 books."

The teacher asked, "Makayla can you show me the 32 books?" Makayla showed the 32 books and counted them verbally.

The teacher said, "Shelby, why do you think she drew 32 books and then 25 books?" Shelby did not provide a clear explanation. The teacher asked Amiah, "Why do you think she drew 32 books and then 25 books?" Amiah said that Makayla took

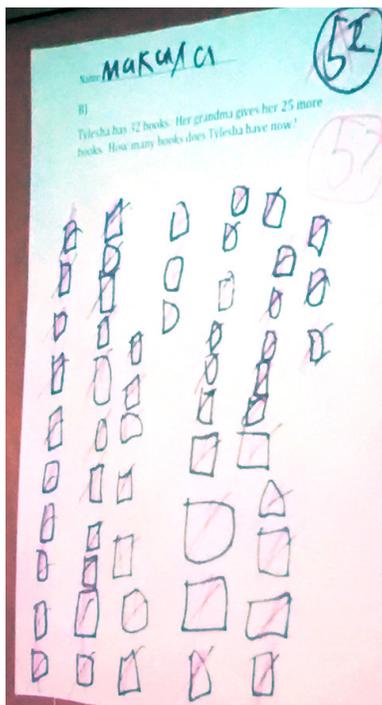


Figure 2. Makayla's written work for the Tylesha's-books problem

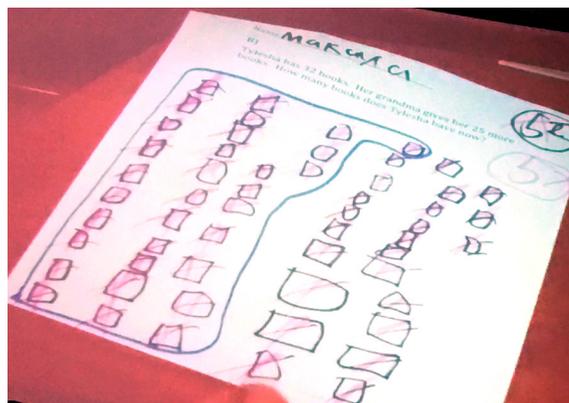


Figure 3. Alexis circled a set of 32 boxes on Makayla's student work

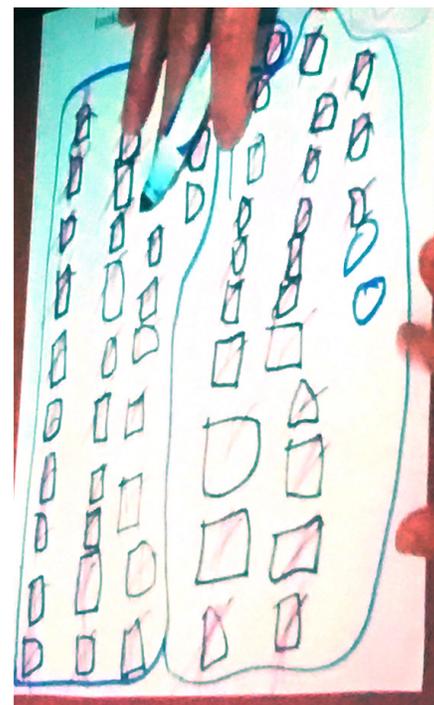


Figure 4. Alexis circled the set of 25 boxes on Makayla's student work

some away.

Elizabeth, another student, said, "No, she added some more books, because you add to get a higher number, and if you take some away, you have less."

The teacher thanked Elizabeth for her explanation. She then turned to Alexis and asked, "Can you show me 32 books on Makayla's paper?" Alexis counted and circled a group of 32 books as shown in Figure 3.

The teacher asked, "Alexis can you count how many are added?"

Alexis counted the remaining shapes that were not circled and discovered there were only 23, and two more needed to be drawn. The 25 circles were circled as shown in Figure 4.

The teacher asked, "Alexis, how many total are drawn?"

Alexis counted by ones—with some correction and support from the teacher—and determined that there were 57.

The teacher asked, "Amiah, what did Alexis do? Could you now finish your problem?" Amiah

agreed, she knew what to do.

The teacher said, "We are now going to see another strategy and compare the two. Wendy, can you bring your paper?" Wendy brought her paper to the screen and used base-ten blocks to represent the two quantities as shown in Figure 5.

Wendy began counting the set of 25 first. She said, "10, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, ... 57." She counted the two unit cubes in 32 last. The teacher placed Makayla's paper next to Wendy's as shown in Figure 6.

The teacher asked, "Makayla can you tell me how Wendy's strategy is different from yours?" Mikayla struggled to answer, so the teacher called on Elizabeth.

Elizabeth said, "It's a little different, because she did it by tens, but they still got the same answer."

While pointing at Makayla's work, the teacher said, "Here is the 32 that Makayla showed in her work. Where is the 32 in Wendy's work?"

The teacher directed the students to turn and talk to each other about how Makayla's strategy and Wendy's strategy are the same and how they are

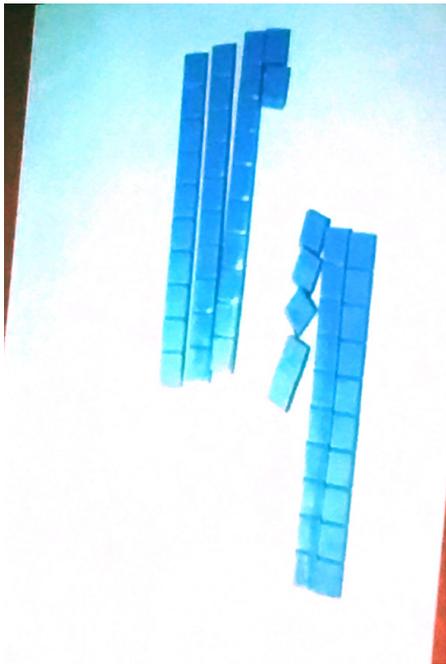


Figure 5. Wendy uses base-ten blocks to represent the two quantities

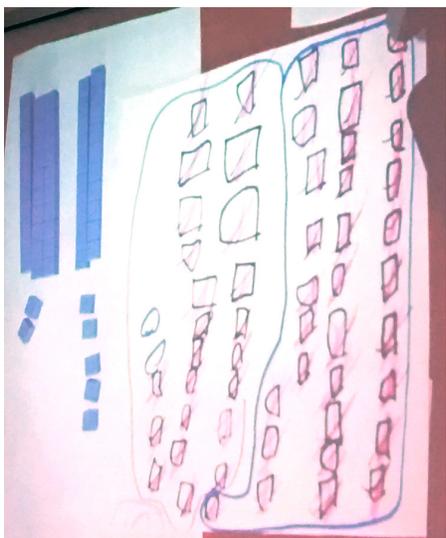


Figure 6. Wendy and Makayla's written work, side by side

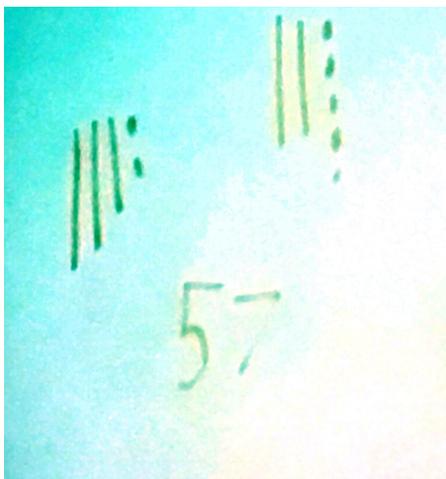


Figure 7. Alyson's student work for the Tylesha's-books problem

different.

The teacher asked, "How are they different? Annaliz can you tell us?"

Annaliz said, "It's because Wendy had tens, and she counted her tens and she counted her ones."

The teacher asked, "What did Makayla do?"

Annaliz said, "She counted her ones."

The teacher asked Makayla, "Where is Wendy's 32?"

Makayla pointed showing where Wendy's 32 was in the representation.

The teacher asked, "Why is this 32?" Makayla said, "There are three tens and two ones."

The teacher asked, "So, all three sticks are 30? Three sticks of ten is going to be what?"

Mikhail said, "10, 20, 30 31, 32."

The teacher asked, "Makayla do you think yours or Wendy's is easier to count? Which one takes longer to count?"

Makayla answered, "Mine would take longer to count."

The teacher said, "Let's try counting them both ways." The teacher counted with the students counting along with her.

The teacher said, "Allyson, I want to see your strategy. Can you show us what you did?"

Alyson brought her work forward as shown in Figure 7.

Alyson said, "I drew 3 tens and 2 cubes. Then, I drew 2 tens and 5 cubes."

The teacher asked, "How did you find the answer of 57? Can you count out loud for me?"

Alyson said, "10, 20, 30, 40, 50, 51, 52, 53, 54, 55, 56, 57."

The teacher asked, "Wendy, did you hear how Alyson counted? How did she count differently than you?" The teacher displayed the representations for both Wendy and Alyson's strategies as shown in Figure 8.

The teacher said, "Wendy, so you counted 10, 20, 30, 31, 32. What did you say after 32?"

Wendy said, "33."

The teacher asked Wendy, "So how did Alyson count differently from you?" Wendy struggled to answer, so the teacher asked Alyson to count again.

Alyson counted, saying "10, 20, 30, 40, 50, 51, 52, 53, 54, 55, 56, 57."

The teacher asked Wendy, "How did Alyson count differently from you?" Wendy was unable to articulate an answer.

The teacher asked Alyson, "Can you show how you put the tens together to count?"

Alyson showed how she put the tens together to count.

The teacher asked Wendy, "How would you count the tens first?"

Wendy said, "10, 20, 30, 31, 32."

The teacher asked Wendy, "What if we save these ones for later?"

Wendy counted and said, "10, 20, 30, 40, 50, 51, 52, 53, 54, 55, 56, 57."

The teacher asked, "So we can count these different ways and still get the same answer?" No student answered, and she left this as a question for individuals to ponder quietly and individually.

The teacher asked, "Jude, can you come tell us

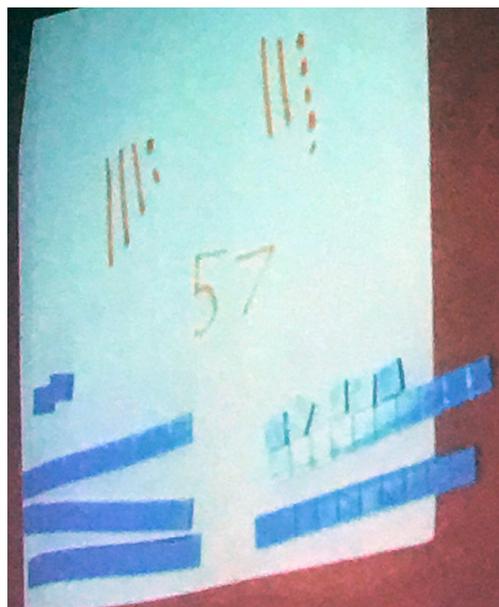


Figure 8. A comparison of Alyson and Wendy's student work

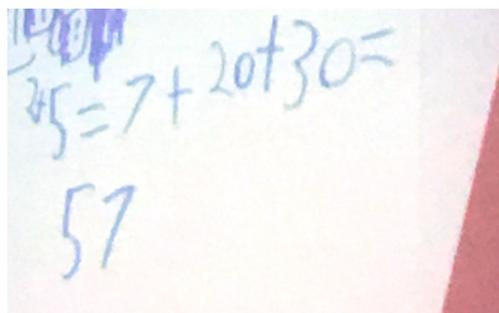


Figure 9. Jude's written work for the Tylesha's-books problem

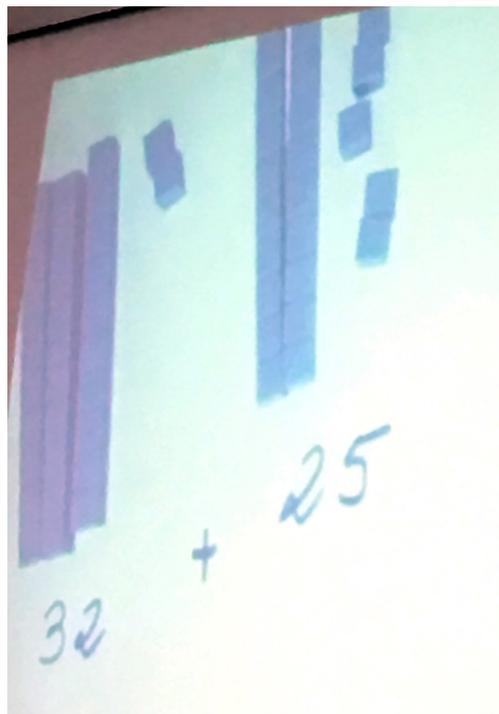


Figure 10. Numerals and the addition operator symbol corresponding to a base-ten-block representation for the Tylesha's-books problem

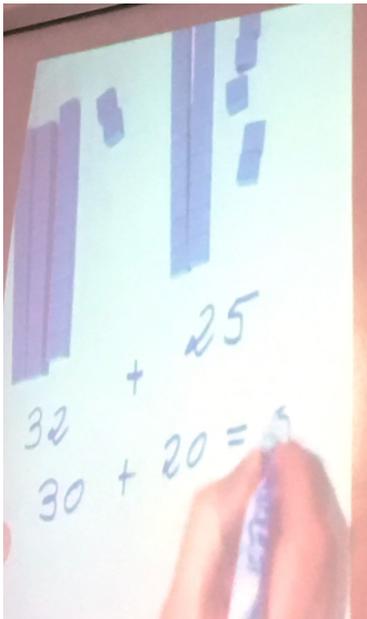


Figure 11. The teacher notates combining the tens for the Tylesha's-books problem

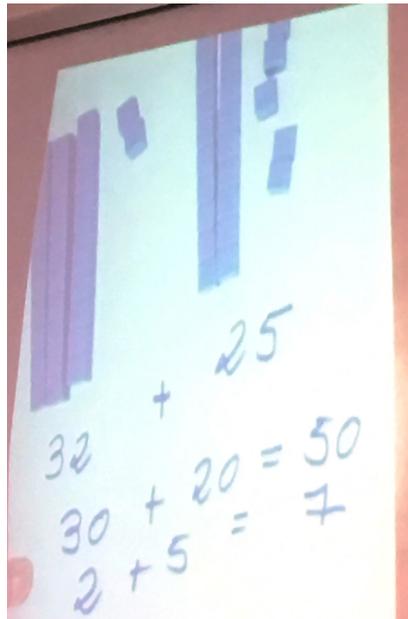


Figure 12. The teacher notates combining the ones

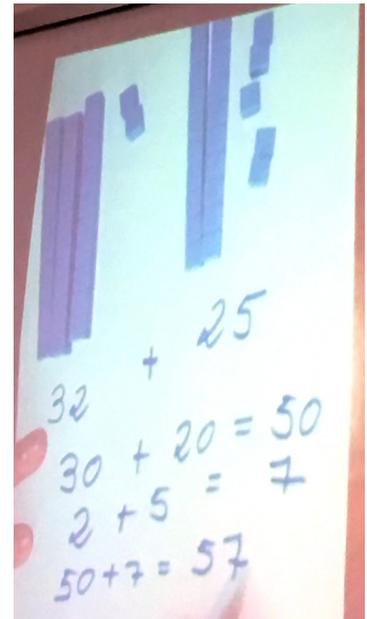


Figure 13. The teacher notates combining the tens and ones for the Tylesha's-books problem

what you did?"

Jude went to the document camera, and said, "I took away the ones and put the tens together.  $5 + 2$  equals 7, and  $30 + 20$  is 50, and then I added the tens and ones together." Jude's strategy was shown to the class using the document camera and is shown in Figure 9.

The teacher asked, "Annaliz, can you tell us what Jude did?"

Annaliz said, "He took away the 2 and the 5 and just added the 20 and the 30. Then he added the 5 and the 2 and got 7. Then he added them together."

The teacher said, "Alyson, you said you counted 10, 20, 30, 40, 50, 51, 52, 53, 54, 55, 56, 57. How is your strategy the same as Jude's? How is it different?"

Alyson said, "He put the 30 and the 20 together."

The teacher asked, "Where did he get the 7?"

Pointing to her paper, Alyson said, "He put these 5 and these 2 together."

The teacher said, "So he took all his tens." I'm

going to write something, and I want you to think about it." The teacher wrote the numbers that correlated to each concrete representation as shown in Figure 10.

The teacher then notated combining the tens as shown in Figure 11.

The teacher then notated combining the ones as shown in Figure 12.

The teacher then notated combining the tens and ones as shown in Figure 13.

The teacher asked, "Can you do this?" Can I just add the ten and the ten? And then add the ones? Talk to your neighbors. Is this allowed?"

Students turned and talked to each other.

After allowing about 30 seconds for discussion, the teacher directed the students to cease talking and come back together as a whole group.

The teacher said, "We have one more problem I want you to solve. Try to use your base-ten blocks or mental math. Try to do something different than you did before to solve this problem." The new problem was posted for students to see. The teacher read aloud the problem and then asked,

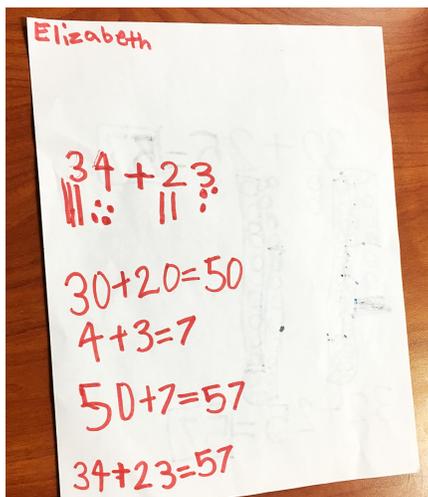


Figure 14. Elizabeth's written work for the new problem

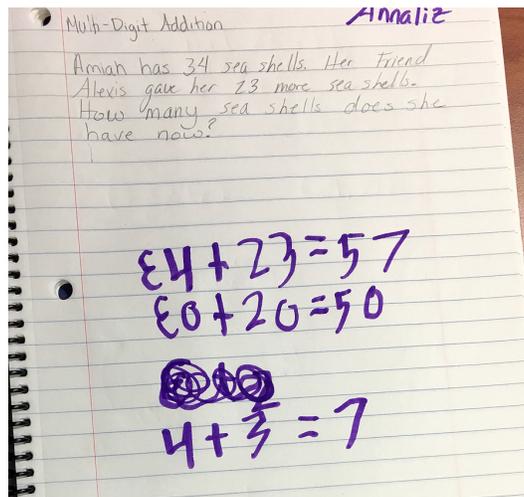


Figure 15. Annaliz's written work for the new problem

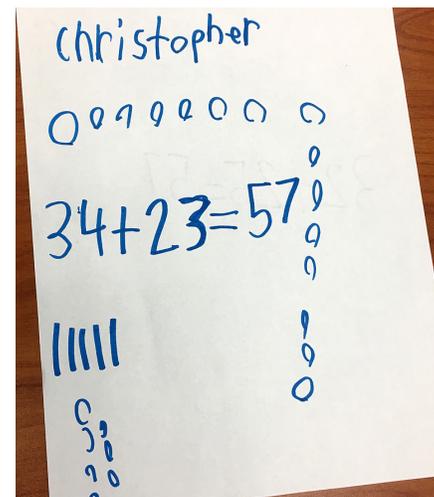


Figure 16. Christopher's written work for the new problem

"Who can tell me what is happening in the story?"

Amiah had 34 sea shells. Her friend Alexis gave her 23 more sea shells. How many sea shells does Amiah have now?

The teacher asked, "What happened in this story? What are trying to find?" After students discussed what was happening in the problem, students were directed back to their seats to solve the problem in a way that differed from the way they solved the Tylesha's-books problem.

The teacher circulated, taking notes of strategies students were using to solve the new problem. The students worked for several minutes, but the teacher had to stop them from working to instruct them to get ready for the end of the school day.

## Reflection

After the lesson, the teachers gathered to discuss and reflect.

The teachers thought that the discussion and learning opportunities during the sharing portion of the lesson were beneficial. They mentioned Nicholas as an example. He was unable to solve the Tylesha's-books problem during the interview. Nicholas did not have an opportunity to speak during the class discussions, so some doubted that he was engaged or benefitted from the lesson. When he was working on the new problem,

the teachers observed him drawing a pictorial representation with tens and ones and then using symbolic notation reflective of a *combining like units* strategy. His work represented a correct solution to the problem. The teachers thought his thinking on the new problem was probably influenced by the experience of watching Alyson and Jude explain their strategies.

During the discussion of the strategies used to solve the Tylesha's-books problem, Wendy changed how she counted and began to group her tens together before counting. On the new problem, Wendy still used a *direct modeling with tens and ones* strategy, but she took the second group of tens, put them with other tens, and counted all the tens and then the ones. When a teacher discretely asked her how she counted the base-ten blocks, she said, "10, 20, 30, 40, 50, 51, 52, 53, 54, 55, 56, 57."

Another example of more sophisticated strategies used to solve the new problem was seen when Elizabeth used a *direct modeling with ones* strategy to determine her answer. On the new problem, Elizabeth drew a pictorial representation with tens and ones and then, like Nicholas, she used symbolic notation reflective of a *combining like units* strategy. Elizabeth's strategy on the new problem is shown in Figure 14.

During the interview, Annaliz direct modeled with tens and ones to determine her answer to the Ty-



Figure 17. Makayla's written work for the new problem

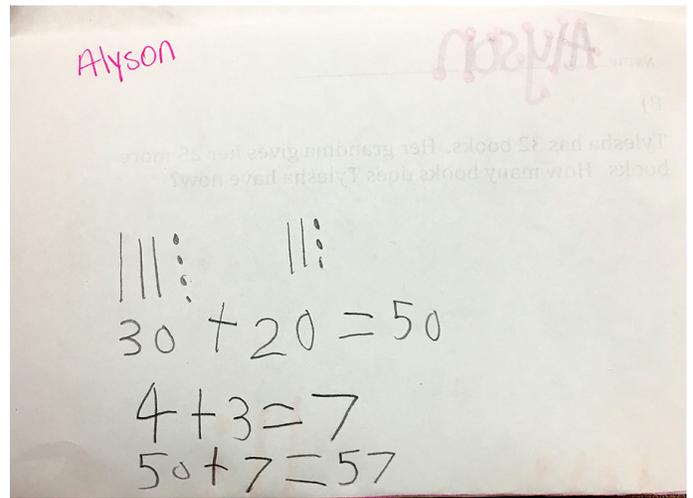


Figure 18. Alyson's written work for the new problem

lesha's-books problem. On the new problem, Annaliz used abstract-symbolic notation and used a *combining like units* strategy. Annaliz's strategy on the new problem is shown in Figure 15.

Christopher used a *direct modeling with ones* strategy on the Tylesha's-books problem. When starting the new problem, he started to use a *direct modeling with ones* strategy again, but one of the teachers intervened and asked he if could solve the problem another way. He then used a *direct modeling with tens and ones* strategy as shown in Figure 16.

Makayla solved the Tylesha's-books problem by using a *direct modeling with ones* strategy. On the new problem, Makayla used a *direct modeling with tens and ones* strategy as shown in Figure 17.

For the Tylesha's-books problem, Alyson used a *direct modeling with tens and ones* strategy. On the new problem, Alyson showed a pictorial representation and used abstract-symbolic notation reflecting a *combining like units* strategy as shown in Figure 18.

In discussion, the teachers agreed that they should have posed a problem that produced a sum different from the one in the Tylesha's-books problem. They thought the next problem to pose to these students might be another join change unknown problem but with the numbers 41 and 53 to determine whether students could think about the

values of the tens and ones places with different values in the tens places. The teachers agreed that most students in this class were ready to solve problems involving numbers that would require them to regroup the ones to make another ten, such as a *join result unknown* problem with the numbers 28 and 56. This problem will probably be more difficult than the problems the students solved today and will further challenge them to think about grouping tens and ones.

The teachers conjectured that students might benefit from more opportunities to share their strategies with the class in the future. They seemed to learn from one another on this day, and their sharing provided good formative assessment opportunities for the teacher. Further opportunities (and expectations) for students to articulate their thinking and to be held accountable for listening to one another might increase both their expressive and their receptive communication abilities. In other words, being expected to share their thinking more often will give them practice explaining their thinking, listening to other ways of thinking, and making connections between different strategies and representations.

# What's Next?

## Stories of teachers engaging in collaborative inquiry focused on using student thinking to inform instructional decisions

*What's Next?* is a collection of stories documenting professional development experiences shared by elementary teachers working collaboratively to study the complex process of teaching and learning mathematics. Each story in the collection describes practicing teachers studying the thinking processes of real students and using what they learn about those students to make decisions and try to help advance those students' understanding on that day.

The teachers in each story start by learning about how individual students are solving a set of mathematics problems. They use this freshly gathered knowledge of student thinking to develop near-term learning goals for students and a lesson plan tailored to specific students on that specific day. One of the teachers implements the planned lesson while the other teachers observe in real time. The teachers then gather to discuss and reflect on their observations and insights.

In these lessons, the practice of teaching is slowed way down. The stories tell of teachers who are studying student thinking and using that information to plan and implement instructional decisions at a pace that is much slower than it occurs in daily practice. The stories in this collection also depict many aspects in common with formative assessment and lesson study, both of which are a process and not an outcome.

The stories depict real situations that occurred in real time and include both successes and shortcomings. We hope that the stories may be studied and discussed by interested educators so that the lessons and ideas experiences of these teachers and instructional coaches may contribute to additional learning and sharing among other interested teachers.

Learn more about these and other stories at <http://www.teachingisproblemsolving.org/>

