Students Moving from Direct Modeling with Ones to Direct Modeling with Tens

This story is a part of the series:

***What's Next?* Stories of Teachers Engaging in Collaborative Inquiry Focused on Using Student Thinking to Inform Instructional Decisions**

What's Next?

Stories of teachers engaging in collaborative inquiry focused on using student thinking to inform instructional decisions

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Introduction

In the context of a professional development experience, a group of teachers collected and used formative assessment data from student interviews to design a lesson on multidigit addition for a first-grade class. The resulting lesson focused on using class discussion of student-generated solution strategies to help advance students' understanding of the base-ten number system.

Relevant Florida Mathematics Standards

MAFS.1.NBT.2.2. Understand that the two digits of a two-digit number represent amounts of tens and ones.

a. 10 can be thought of as a bundle of ten ones, called a "ten."

b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.

c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).

d. Decompose two-digit numbers in multiple ways (e.g., 64 can be decomposed into 6 tens and 4 ones or into 5 tens and 14 ones).

MAFS.1.NBT.3.4. Add within 100, including adding a two-digit number and a one-digit number and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a "new" ten from some of the ones.

Background Information

To understand this lesson better, consider becoming familiar with frameworks describing how children think about and solve addition problems and how they develop base-ten number-system understanding through working with multidigit numbers. An introduction to typical strategies children use to solve addition and subtraction problems and how those strategies evolve as children develop more sophisticated knowledge and understanding appears in chapter three of *Children's Mathematics: Cognitively Guided Instruction* (Carpenter et al., 2015). Chapter four of the same book provides a description of how problems with two- and three-digit numbers can be used to help children develop understanding of base-ten number concepts.

Carpenter, T., Fennema, E., Franke, M., Levi, L., & Empson, S. B. (2015). *Children's Mathematics: Cognitively Guided Instruction (2nd ed.)*. Portsmouth, NH: Heinemann.

Analyzing Student Thinking

A group of teachers studying students' mathematical thinking posed several word problems to students in one-on-one interviews. Students were informed that the purpose of the interviews was for the teachers to learn how these students were thinking about mathematics and what they knew about numbers. Each student was interviewed individually by a pair of teachers who asked them to explain their thinking. The teachers took notes on the students' problem-solving process. The children were asked to solve four word problems. Three of the problems were *join result unknown* problems, and one was a *separate result unknown* problem. *Join result unknown* word problems involve a change in quantity that occurs over the course of the story in the word problem, and a known starting quantity is increased (additively) by a known particular amount; the resulting quantity is unknown and is the focus of the question the problem solver is trying to answer. Problems A, B, and D in the following set of problems are classified as *join result unknown* problems. Sim-

They also noticed that asking questions about how strategies are similar and how they are different may have helped students notice that two-digit numbers can be represented in different ways.

ilarly, *separate result unknown* word problems involve a change in quantity wherein the starting quantity is given and decreases by a known amount over the course of the story. Problem C is an example of a *separate result unknown* problem type.

Problem A
Pete had 20 rocks. Juan gave him 24 more rocks. How many rocks does Pete have now?

Problem B
Tylesha had 32 books. Her grandma gave her 25 more books. How many books does Tylesha have now?

Problem C
Mr. Jones had 40 cupcakes. He gave 20 cupcakes to the students in his class. How many cupcakes does Mr. Jones have now?

Problem D
Maria had 35 jellybeans. Her dad gave her 27 more jellybeans. How many jellybeans does Maria have now?

The group of teachers first analyzed the interview tasks and anticipated strategies that students would generally use to solve the four word problems. They asked themselves, "What would a first-grader do to solve each of these problems?" They discussed what was similar and what was different about each of the problems and mostly focused on the structure of the problems and the numbers involved in the problems. The teachers noticed the result, or the final amount, was the unknown quantity in each of these four problems. They also observed that Problem C was a *separate* word problem, and Problem D differed from the first two *join* problems, because finding the answer to problem D involved regrouping. The teachers focused their attention on predicting strategies demonstrating that students are understanding important base-ten concepts such as organizing linking cubes into groups of ten, organizing by means of a ten-frame arrangement, or adding all the base-ten rods representing tens and separately adding the base-ten unit cubes representing the ones. When discussing which problem might present these first-graders with the most difficulty, the teachers predicted it would be Problem D, because it required regrouping. There was some discussion that the same problem, however, may be easier for students who counted by ones to solve the problem. (See below for a description of student strategies.)

During the interviews, the teachers planned to read one problem at a time out loud and allowed the students significant time to think, model, write, and explain their solutions. Teachers would refrain from telling or guiding the children to solve problems in certain ways, because they were using the interview to learn about the children's mathematical thinking. Telling children what to do or guiding them to do certain things would not yield the same type of assessment data. The interview was not intended to be a tutoring session. Rather, it was an opportunity to learn about the child's under-

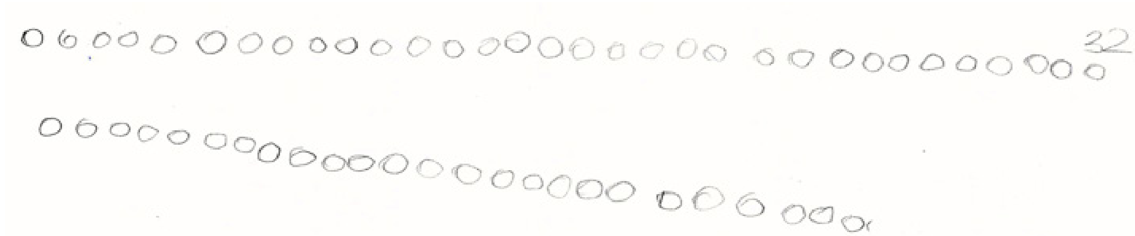


*Figure 1. A representation of a **direct modeling with ones** strategy for the **Tylesha's books** problem.*

standing of mathematical ideas. Students were offered linking cubes, base-ten blocks, and writing materials as tools for problem solving. The teachers used various probing questions such as "How did you solve that?" "When you solved the problem, what did you do first?," and "I saw you counting those cubes silently. Can you tell me what you were hearing in your mind as you were counting?" After solving each problem, students were encouraged to write a number sentence for the completed problem. Teachers also asked students how many linking cubes were in the provided rods and try to determine through observation and questioning whether the students treated each rod as a ten or as ten units of one.

At the end of the interviews, which took about 15 minutes each, the teachers studied the student responses and sorted them into groups according to the strategies observed. As a group, they chose to analyze and sort student thinking around Problem B (i.e., the Tylesha's-books problem), because the teachers agreed that it appeared to show the students' typical way of thinking. The following categories of student strategies were used.

Multidigit Computation Strategies[1]

A student who uses a *direct modeling with ones* strategy represents each multidigit number in the problem as a set of ones using manipulatives or pictures to model the story in the problem and then counts the objects or pictures to determine the answer. Figure 1 is an example of work from a student using such a *direct modeling with ones* strategy to solve the Tylesha's-books problem. The student drew 32 circles in a row to represent Tylesha's original books. She then drew 25 circles underneath to represent the books Tylesha received. Although she wrote 32 at the end of the first row of circles, she still counted all the circles again to arrive at her answer, 57.

Some variation on this strategy was observed, as shown in Figure 2, which the teachers considered more advanced thinking than that presented in Figure 1. The student modeled the numbers using base-ten understanding and counted the first number by ten but the second number by ones (10, 20, 30, 31, 32, 33, 34, 35, 36… , 57).

A student who uses a *direct modeling with tens* strategy represents each multidigit number in the

1 The descriptions of strategies presented here are the current descriptions used by our team, and we consider them to be fluid, as our understanding of these ideas continues to evolve. For a more detailed discussion of these terms, consider reading Carpenter et al. (2015).

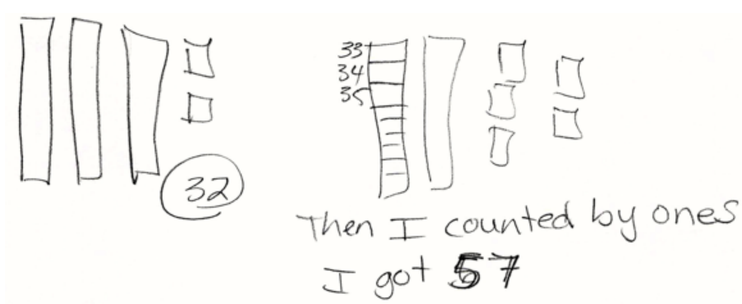


Figure 2. A representation of a direct modeling with ones strategy for the Tylesha's-books problem. This is a photograph of the teacher's notes, which were a replication of the child's drawing along with numerals and words to record what the child said.

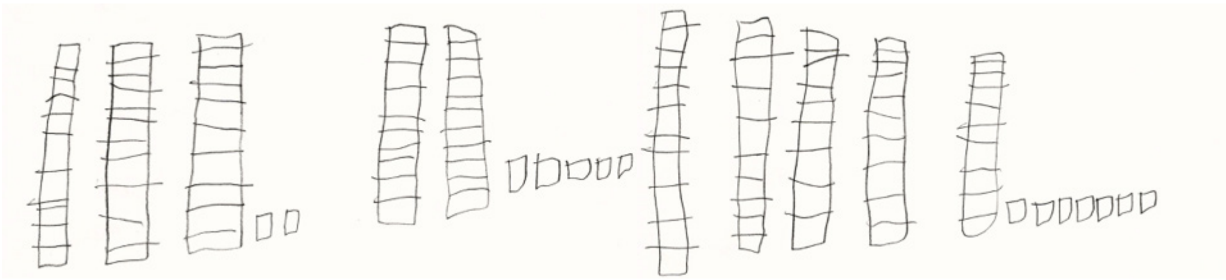


Figure 3. A representation of a ***direct modeling with tens*** *strategy for the Tylesha's-books problem.*

problem using manipulatives or pictures that reflect the base-ten structure of the number system (e.g., with base-ten blocks or base-ten pictures). Then, the student counts the objects or pictures by tens and ones to determine the answer. A typical example of this strategy is shown in Figure 3, where the student very clearly modeled the two quantities using base ten understanding and then drew how he separated the tens and ones and counted by tens: 10, 20, 30, 40, 50, 51, 52, 53, 54, 55, 56, 57.

A student who uses an *incrementing* strategy works with numbers more abstractly than students who use direct modeling strategies and determines the answer by increasing or decreasing partial sums or differences. In the example problem, the student might start at 32 and then decompose the 25 so it could be added more easily in increments of tens and ones. Using this approach, the student might say or write 32, 42, 52 to account for the tens in 25, then add the ones still left: 53, 54, 55, 56, 57. None of the interviewed students used this strategy.

A student who uses a *combining the same units* strategy operates on the tens and ones separately and then combines partial sums to get a final result. This is an internalization of the *direct modeling with tens* strategy shown in Figure 3, but students who use an invented algorithm do not need to create a physical or pictorial model of the quantities to find the answer. Instead, they are able to use knowledge of number relationships to decompose the quantities into partial sums in their minds or with numbers written on paper. Figure 4 is an example of how a student wrote an equation to represent how he calculated the problem in his head.

The teachers organized the student work by the strategies used for the Tylesha's-books problem (Figure 5). Two of the students were not able to correctly solve the problem as it was given. Angelina had problems with story comprehension and subtracted the two numbers. She drew 32 circles, and took away 25 by crossing out 25 of the circles. Tabitha understood the story but had difficulty working with such large numbers. After observing this, the interviewer asked her to solve the same

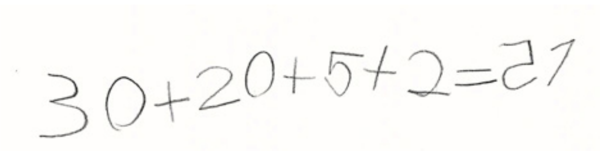


Figure 4. Symbolic representation written by a student who used a ***combining the same units*** *strategy.*

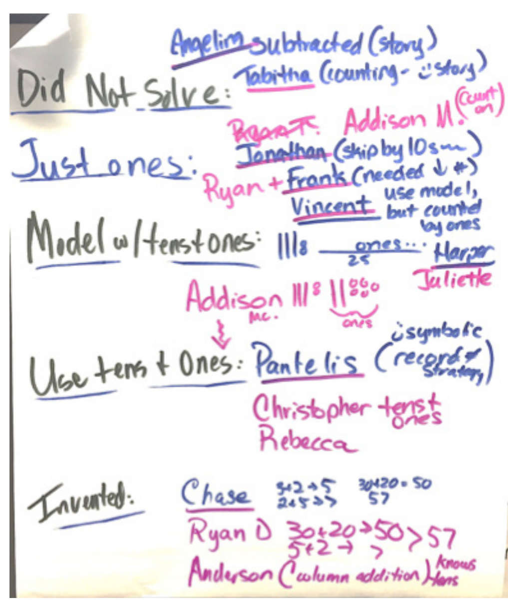
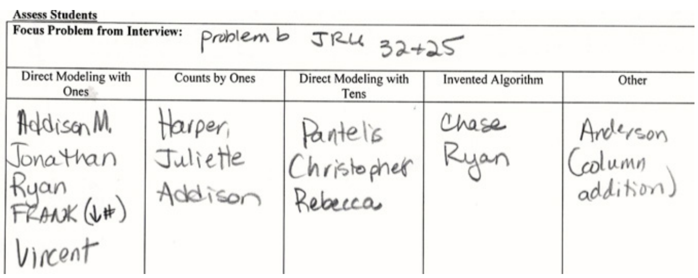


Figure 5. A chart showing names of students organized according to the strategies they used to solve the Tylesha's-books problem.

problem with smaller numbers, and she was able to solve the problem correctly and easily with the replacement numbers. The rest of the students' strategies were sorted according to the previously mentioned strategies. Figure 6 shows the final grouping of students on the basis of strategies used.

Of the students classified under *direct modeling with ones*, Frank needed smaller numbers to solve the problem successfully. Addison represented 32 with tens and ones, but he represented 25 as a collection of ones and *counted on by ones* from 32. When Jonathan began the problem he represented 32 with three tens and two ones and represented 25 as two tens and five ones. And


Assess Students

Focus Problem from Interview: Problem b JRU 32+25

Direct Modeling with Ones	Counts by Ones	Direct Modeling with Tens	Invented Algorithm	Other
Addison M. Jonathan Ryan FRANK (↓#) Vincent	Harper, Juliette Addison	Pantelis Christopher Rebecca	Chase Ryan	Anderson (column addition)

Figure 6. The final sorting of students after some discussion.

when he counted these numbers individually, he counted by tens and ones. But when finding the total, he counted by tens once and then counted the remaining 57 cubes by ones. This sounded like, "10, 20, 21, 22, 23, 24, 25, 26, 27…57."

The students in the intermediary stage who modeled one or both numbers with tens and ones but then counted by ones or counted on from the first number by ones were grouped under *modeling with tens and ones* and then under *counting by ones*. Juliette modeled with tens and ones but ignored the set up and counted by ones to find the answer. In Figure 2, Addison modeled with tens and counted by ones to get to the second number.

The list of students in the *direct modeling with tens* category included students who both modeled with tens and ones, and who also counted with tens and ones. Christopher's work is shown in Figure 3.

Two students did not make use of manipulatives or models but solved the problem using invented algorithms[2], and they both *combined the same units*. Chase wrote 57 after hearing the problem. He explained that he knew that 3 and 2 is 5, and 5 and 2 is 7. He then explained that he knew the answer was 57, because the 3 is 30 and the 2 is 20. Ryan also produced the answer very quickly and explained that 30 + 20 = 50 and 5 + 2 = 7 (in that order). His written explanation is shown in Figure 4.

One final student, Anderson, was placed in the "other" category, because he used column addition to carry out the U. S. standard algorithm for addition to find the answer. The teachers who interviewed him decided that he had a good understanding of place value and was using column addition with understanding of place value.

Setting near-term learning goals for these students

After examining the ways students in the class were thinking about numbers, the teachers were surprised and pleased to see the variety of strategies already used by students, because this was the first time these students had encountered problems involving multidigit numbers in first-grade. The teachers decided to develop a lesson which made use of students' existing understanding of tens to try to deepen their understanding of the base-ten number system. In their solution strategies, almost all of the students had used groups of tens in some way. The teachers wanted students to see connections between direct modeling of problems involving single-digit numbers and direct modeling of problems involving two-digit numbers. They also hoped to be able to connect the direct modeling strategies with some of the *ad hoc*, numerically-specific strategies that Chase and Ryan had used. They wrote the following goals for the upcoming lesson.

1. Students will notice and discuss base-ten understanding.

2 This term is used in a way similar to the way it is used in Carpenter et al. (2015). The use of the term algorithm here is not intended to imply that the strategy could or would be repeated in a generalizable form.

2. Students will be able to use direct modeling with tens *consistently.*

Planning for the Lesson

Because a good range of ideas was observed with the Tylesha's-books problem, the teachers decided to start the lesson by revisiting those strategies and making connections across them, and then presented a new problem which would help students apply the ideas they discussed in the first part of the lesson.

Part 1: Discussion of strategies students used solving the Tylesha's-books problem

Tylesha had 32 books. Her grandma gave her 25 more books. How many books does she have now?

Part 2: New mathematical task asking students to use working base-ten understanding

Abby got 16 Valentines cards at school. Then she got 23 more cards from her afterschool group. How many cards did she get in all?

Possible challenge task idea: Replace the original numbers with (30, 17) in the same problem.

Rationale for the new problem selected

The teachers chose to create a lesson around the two problems to bring students' existing notions about the relation between tens and ones to the forefront. They also wanted to create a discussion about how to use tens and to allow for both simple and more sophisticated ways to think about place value ideas.

The Tylesha's-books problem was chosen as a platform for the discussion, because the students used the widest variety of strategies to solve it during the initial interviews. The teachers also created a plan to structure the discussion and deliberately chose the students who would share their strategies with the rest of the class. They chose students who had good pictorial work and used tens and ones to solve the problem, and they ordered their presentation from the most concrete and least sophisticated, to the most abstract ways of thinking about the problem.

The new word problem for Part 2 was related to the time of year during which it was presented (i.e., shortly after Valentine's Day). They elected to pose an addition problem, because most students were successful with addition in the interviews, and they wanted to be able to focus on place value concepts. To this end, they decided to use a *join result unknown* problem. The numbers 23 and 16 were selected for the problem, because the teachers wanted to offer an entry point for students who were still counting by ones and those who might have trouble with larger numbers while still providing opportunity for the more advanced students to use their ideas about place value to help them solve the problem more efficiently. In addition, the teachers wanted to use multidigit numbers that made sense within the context of the problem. They decided that it was plausible to get 16 cards from other students in the class and to get more than that in their after school programs, where there might be more children in the group. Finally, the teachers wanted to make sure the numbers chosen would not require regrouping, because the children were just starting to work with such large numbers.

Strategy for differentiation to meet the needs of all students in the class

For Part 1 of the lesson, the teachers decided to plan the sequence of students' strategies to be shared in class discussion based on the strategies students used in the interviews. They planned to orchestrate a discussion around the use of tens and ones to solve the problem, because they conjectured that it would allow all students to participate in the discussion regardless of their level of understanding of base-ten concepts.

Figure 7 shows the plan the teachers made for the discussion and the progression of student thinking, from *direct modeling with ones* to more abstract strategies.

The first student chosen to share her work, Addison, used a *direct modeling* strategy and counted by ones (Figure 1). She was chosen to share, because a good record of her thinking was available for the other students to observe. Teachers planned to have her count out loud, either by herself or possibly with the entire class.

The second student chosen to share was Rebecca. She used a *direct modeling with tens* strategy but counted on from 32 by ones. (See Figure 2.) She used physical manipulatives and drew a pictorial representation of them on paper. The class could be asked to predict how she counted from looking at her paper before she was asked share her strategy. If one of the students miscounted by one, for example, teachers planned to compare the two strategies and discuss the differences in answers.

Christopher was chosen to share next, because he modeled with tens and ones and counted by tens and ones, and he had a highly organized and complete model (Figure 3). He also used both physical manipulatives and a drawing. The teachers planned to have the rest of the class make conjectures about how Christopher solved the problem and to compare his work to the strategies of the other two students (who had counted by ones).

Finally, Pantelis was chosen to share his work because of the interesting symbolic representation he wrote on his paper (Figure 8). He used a direct modeling with tens strategy and also wrote a number sentence based on his strategy that could be used to make the connection between direct modeling and invented algorithms.

The teachers planned to draw the students' attention to the similarities among the various strategies shared in class discussion. They also planned to present two strategies at a time, side by side, to make connections and compare differences. In addition, the teachers decided that they wanted the class discussion to emphasize counting by tens. They planned to direct this emphasis, in part, by having the entire class count out loud, especially with those students who used a *direct modeling with tens* strategy.

The problem chosen for the second half of the lesson had smaller numbers than did the Tylesha's-book problem. They were chosen intentionally to help engage students like Tabitha and Frank, who were successful in solving the problem with smaller numbers, but struggled with 32 and 25.

Strategies to be shared:	Teacher moves when this strategy is shared	Notes during the lesson
Addison M Counts by ones, models by ones	Count out loud ask students to predict, talk about conserving	the first number
Addison model, count on by ones	Count 10's + ones predict how recount when arrive at 58	Compare two Strategies
Christopher nice model + count	Show pic. predict, have him explain compare	
Pantelis numbersentence	ask to solve own way, then match to the question strategy	

Figure 7. Student sharing plan for strategies discussion on the ***Tylesha's books*** *problem*

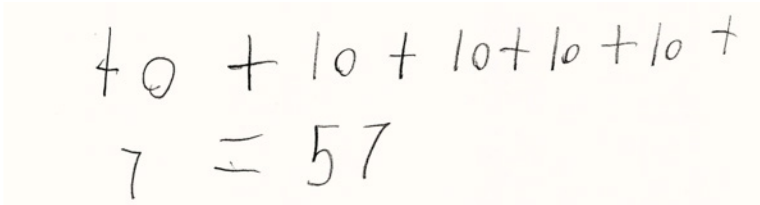


Figure 8. Student who added the tens and ones separately and recorded his thinking with an equation.

Lesson Plan

While planning for the lesson, the teachers developed the following learning goals for students:

1. Students will notice and discuss base-ten understanding.

2. Students will be able to use direct modeling with tens *consistently.*

Preparation for the Lesson

Before the lesson, conduct initial interviews and sort student work according to the aforementioned criteria. Choose students who will share their work and the sequence in which they will do so. Make available the same physical manipulatives used during the initial interviews so that students can recreate their work for the class. Have a document camera ready to display the student work.

Part 1: Discussion of strategies students used solving the Tylesha's books problem

1. Remind students of the problems they solved during the initial interviews, and inform them that you are going to be working together to examine some of the ways students in the class solved one of those problems. Read the Tylesha's-books problem aloud to class.

Tylesha had 32 books. Her grandma gave her 25 more books. How many books does she have now?

2. Ask probing questions to ensure that students recall and understand the details of the problem. For example, read the first sentence and ask, "What happened after that?" Allow students to reply. Ask students, "What did you need to find out?"

3. Invite students to explain the important details of the problem to a shoulder partner.

4. Invite a student who used *direct modeling with ones* during the interviews to display his or her solution (on a document camera, on the board, or on the carpet). Help the class to understand the students' strategy:

a. Ask the class, "What do you see? What do you think ____ did to solve the problem?"

b. Ask the student who generated the strategy to verify or refute conjectures made by the class and/or explain the strategy.

c. Ask the student to explain how he or she solved the problem. If the student has trouble remembering, gently ask probing questions as a gentle memory aid.

d. Ask the student to count out loud just as they did when solving problem and ask the class to join in counting out loud with the student.

5. Repeat the process with a student who counted on from the first number to display his or her solution (on a document camera, on the board, or on the carpet). Help the class to understand the student's strategy:

a. Ask the class, "What do you see? What do you think ____ did to solve the problem?"

b. Ask the student who generated the strategy to verify or refute conjectures made by the class and/or explain the strategy.

c. Ask the student to explain how he or she solved the problem. If the student has trouble remembering, ask probing questions as a gentle memory aid.

6. Place the two strategies side by side and have students look for similarities and differences.

a. Ask the class, "What do you notice? How are these two strategies the same?" Allow sufficient time for students to think and answer.

b. Ask the class, "How are these two strategies different?" Again, allow time for students to answer. Then say, "In the second strategy I see that I don't have to count the first number. I can start at 32."

c. Ask a student from the audience to come and point to the objects while counting out loud from 32. Ask all students to count out loud in chorus while student at the front counts and touches each circle, "33, 34, 35…" Ask, "Why don't you have to count the first group?" If the student also used manipulatives, ask the student to use them again to recreate the problem-solving process under the document camera.

7. Ask a student who used *direct modeling with tens* to come and share his or her work. Place the second and third strategies side by side under the document camera.

a. Ask the class, "What do you notice? How are these two strategies the same?" Allow sufficient time for students to think and answer.

b. Ask the class, "How are these two strategies different?" Again, allow time for students to answer. Draw students' attention to how the third student counted by ten.

c. Ask the class to count out loud with the student using tens and ones.

8. If a student used a written equation, ask that student to present his or her work.

a. Ask the class to look at the number sentence.

b. Take the base-ten rods that the previous student used and add a rod over each ten and the ones over the 7 in the number sentence. Ask the students if they understood what you did.

c. Ask students to explain the action in their own words.

9. Conclude the segment by asking the class, "Did you learn anything new?"

Part 2: New mathematical task asking students to use working base-ten understanding

1. Present the new problem to the class and use strategies to support students' comprehension.

a. New problem: *Abby got 16 Valentines cards at school. Then she got 23 more cards from her afterschool group. How many cards did she get in all?*

b. Possible strategies to support comprehension:

i. Have students discuss the important details of the problem with a shoulder partner;

ii. Ask the class questions to probe understanding of the problem. For example, "What happened first? What happened next?"

c. Direct students to think of a strategy for solving problem and to give you a signal (e.g., thumbs up on chest) to indicate when they have an idea.

i. Once most students show that they have strategies in mind, ask them to go back to their desks and start solving the problem.ii. Remind students they can use whatever tools they want to solve it and whatever method they feel most comfortable with, but emphasize the use of blocks first before they start to draw.

2. Circulate and observe what students are doing. Allow enough time for students to solve the problem in whatever way they choose and to record their thinking in numbers, pictures, or words. Attend to the following:

a. Do students use a strategy similar to or different from the one they used on the previous problem?

b. For those who model the problem with objects or pictures, do they choose to model and count by tens, when possible?

c. Do students notate their strategy symbolically (e.g., with a number sentence)? If yes, consider noting how.

3. Ask questions to support student understanding of the problem and to improve your understanding of their strategies:

a. For students who do not appear to be actively working on the problem, you may want to diagnose whether they understand the context of the problem. To do this, you might say, "Tell you tell me what you know about this problem."

b. For students who are actively working on a strategy, quietly observe them to gain insight into their cognitive processes. If you can't ascertain how they're solving the problem during your observation, you may choose to prompt them to tell you how they are thinking. An example of a prompt might be, "Tell me about what you're doing to try to solve this problem."

4. Identify 2–4 student strategies to be shared in class discussion that will work toward your instructional goals (sharing could occur during this lesson or at the beginning of the next lesson). One possibility is to choose at least one student who modeled with tens and counted with tens, so that this strategy can be further reinforced.

5. Gather the whole class and establish the purpose of the discussion: "To encourage students to notice and discuss what is the same and what is different between how they solved the problem and the shared strategies."

6. Invite selected students to share their strategies one at a time. Consider sequencing strategies in order of sophistication.

a. Ask students to use base-ten rods and match them with the drawing and number sentences if they have not done so.

b. Ask them to count out loud, especially if counting by tens. Ask the rest of the class to join them in counting. When the students put the rods and ones together, ask the class, "Where is the 23? Where is 16?"

c. Place the different strategies next to each other and ask the class, "How are these strategies different and how are they the same?"

7. End the lesson by asking students to share with a partner one idea they learned today. Listen to what they say. You may want to have a few share what they learned out loud.

Reflection

What we have learned about the students

The lesson was helpful in getting students to notice tens and to make connections between *direct modeling with ones* strategies and *direct modeling with tens strategy*. In the initial interviews and the lesson, the teachers noticed that asking students to make predictions and to guess what students did in their strategies may have helped students to make connections between strategies on their own. They also noticed that asking questions about how strategies are similar and how they are different may have helped students notice that two-digit numbers can be represented in different ways. Some teachers noticed that students with different models, either by ones or by tens, both counted on from the first number.

After the discussion in the first half of the lesson, some teachers noticed that students started to use strategies and techniques presented by other students. Some students were using other students' modeling ideas and writing numerals next to the models.

Because the lesson was separated into two parts, and the discussion about student strategies lasted a bit longer than anticipated, there was very little time left for the planned discussion at the end of the new task. In the future, planning for a carefully chosen class discussion on a specific student strategy or pair of strategies would probably be a better way to organize the lesson.

The teacher noticed that some students showed inconsistencies between how they modeled using manipulatives and their model on paper. For example, Figure 9 shows the work of a student who modeled 23 using two rods and three ones and modeled 16 with one rod and six ones. When drawing a model, however, the student used ones even though they were grouping them in lines of ten. Although it was not done in the lesson, teachers thought that a discussion about the difference between the student's model and picture can be turned into a valuable teaching moment during the class discussion. This piece of student work can serve to generate discussion about how grouping by tens is done in the base-ten rods, the picture, and the equation.

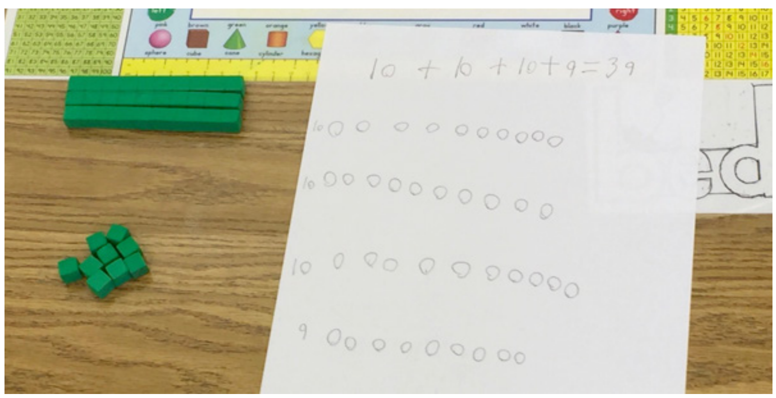


Figure 9. A student's solution to the ***Abby's Valentine Cards*** *problem*

What's Next?

Stories of teachers engaging in collaborative inquiry focused on using student thinking to inform instructional decisions

What's Next? is a collection of stories documenting professional development experiences shared by elementary teachers working collaboratively to study the complex process of teaching and learning mathematics. Each story in the collection describes practicing teachers studying the thinking processes of real students and using what they learn about those students to make decisions and try to help advance those students' understanding on that day.

The teachers in the stories start by learning about how individual students solve a set of mathematics problems. They use this freshly gathered knowledge of student thinking to develop near-term learning goals for students and a lesson plan tailored to specific students on that specific day. One of the teachers implements the planned lesson while the others observe in real time. The teachers then gather to discuss and reflect on their observations and insights.

In these lessons, the practice of teaching is slowed way down. The stories tell of teachers who are studying student thinking and using that information to plan and implement instructional decisions at a pace that is much slower than is typical in daily practice. They depict a process with many aspects in common with formative assessment and lesson study, both of which are also conceptualized as processes and not outcomes.

The stories depict real situations that occurred in real time and include both successes and shortcomings. We hope they will be studied and discussed by interested educators so that the lessons and ideas experienced by these teachers and instructional coaches will contribute to additional learning and sharing among other interested teachers.

Learn more about these and other stories at **http://www.teachingisproblemsolving.org/**