Students Make Sense of Problems Involving Groups of Ten

This story is a part of the series:

***What's Next?* Stories of Teachers Engaging in Collaborative Inquiry Focused on Using Student Thinking to Inform Instructional Decisions**

What's Next?

Stories of teachers engaging in collaborative inquiry focused on using student thinking to inform instructional decisions

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Introduction

In a year-long professional-development experience, a group of teachers interviewed students in a second-grade class. On the basis of these interviews, they developed a lesson that encouraged students to share their strategies for solving a word problem involving multidigit numbers. The lesson is intended to help students make connections among their strategies so as to improve students' place-value understanding.

Relevant Florida Mathematics Standards

MAFS.2.NBT.1.1 Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases:

a. 100 can be thought of as a bundle of ten tens—called a "hundred."

b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).

Background Information

Our investigation was strongly informed by chapters six, seven, and nine in *Children's Mathematics: Cognitively Guided Instruction* (Carpenter et al., 2015). Chapter six provides background information on base-ten number concepts. Chapter seven provides information on children's strategies as they solve problems involving multidigit numbers. Chapter nine provides information on how to pose problems and elicit students' thinking.

In addition to that book, chapters three and four in *Young Mathematicians at Work: Constructing Number Sense, Addition, and Subtraction* (Fosnot & Dolk, 2001) may be a good source of additional ideas. Chapter three provides background information on number sense, and chapter four provides information about place value.

Carpenter, T. P., Fennema, E., Franke, M. L., Levi, L., & Empson, S. B. (2015). *Children's Mathematics: Cognitively Guided Instruction (2nd. Ed.).* Portsmouth, NH: Heinemann.

Fosnot, C. T., & Dolk, M. (2001). *Young Mathematicians at Work: Constructing Number Sense, Addition, and Subtraction*. Portsmouth, NH: Heinemann.

Analyzing Student Thinking

Three mathematics problems were posed to second-grade students near the beginning of the school year. A group of teachers studying addition and subtraction word problems and strategies for solving them interviewed each student in one class individually, to gather information about each child's mathematical thinking and understanding on that day. The three problems were created and sequenced in an attempt to provide insight into how these students approached addition and subtraction problems. The interviewers were allowed to skip any problem they thought was too challenging for the student being interviewed. Each student had access to linking cubes, base-ten blocks, pencils, and paper during the interview, and they were encouraged to use whatever method made the most sense to them. Each item was read aloud to each student by the interviewer and reread as needed. After the student provided an answer, the interviewer asked the student to explain the strategy he or she used to solve the problem. The following problems were posed to the students.

> *Problem A. I have 5 boxes with 6 cookies in each box. How many cookies do I have?*
>
> *Problem B. I have 8 boxes of pencils with 10 pencils in each box. How many pencils do I have?*
>
> *Problem C. The second graders at Orange Grove Elementary School raised $67 to buy books for the children's hospital. If each book cost $10, how many books can they buy?*

The following section provides a summary of the working definitions of student strategy categories that are discussed below.[1]

Strategies Typically Used by Students to Solve Problems Involving Multidigit Computation

The student who uses a direct modeling strategy represents each quantity in the problem concretely—typically by using physical objects (including fingers) or by drawing representations of them on paper—and models the story in the word problem scenario. To solve problem A, a student using a *direct modeling* strategy might draw five boxes with six cookies in each box and would count all the cookies by ones, by skip counting, or by some combination of the two methods.

The student who uses a *direct modeling by ones* strategy represents all the quantities in the problem (grouping ten in each group) and counts by ones to determine the solution. In problem C, a student using a *direct modeling* strategy might draw 67 objects and circle groups of ten objects. Then the student might count the groups of ten to determine the answer.

The student who uses a *direct modeling by tens* strategy represents all the quantities and counts by tens and ones, keeping track with manipulatives or drawings (including fingers). A student using this strategy to solve problem B might draw eight groups with ten in each group. Then the student might count, "ten, 20, 30, 40, 50, 60, 70, 80" to determine the answer (rather than counting by ones). Of course, some students might use a combination of these strategies. For example, they might count by tens to 30 and then by ones from 31 to 80.

The student who uses a *counting* strategy does not represent each and every object in the problem physically but does count by ones or by skip counting (rather than by using recalled facts). In problem A, a student using a *counting* strategy might skip count by sixes and say, "6, 12, 18, 24, 30," holding up one finger for each number verbalized. After raising five fingers, the student knows to stop counting.

A *fact recall—known fact* strategy involves the student's recalling the relevant fact(s) directly from memory. When these student are asked to explain how they got their answers, they often respond that they "just knew" the answer or "because six times three just equals 18," or "because I did that one before." Asked how he or she found the answer of 80 for problem B, for example, might respond "I just know" the answer or "because I know eight times 10 is 80."

The student who uses a *direct place value* strategy knows how many tens and ones are in the given number and provides the answer without drawing or creating a physical representation. In problem C a student using this strategy might explain, "six, because there are six tens in the number 67."

Strategies Used by Students in This Second Grade Class

After the student interviews were completed, the teachers classified the students according to the strategies they used to solve each problem. The teachers observed that most students were successful in solving problems A and B. As Figure 1 shows, many students did not correctly solve Problem C. The sorting categories for problems B and C are the same, because students could solve the problems using similar strategies. Problem A has different categories of strategies, because the numbers in the problem are unlikely to result in students' using a *direct place value* strategy.

The teachers observed that about half of the students did not understand problem C and did not use a viable strategy to solve it. They therefore developed the following learning goal for these students on this day:

1 The descriptions of strategies presented here are the current descriptions used by our team, and we consider them to be fluid, as our understanding of these ideas continues to evolve. For a more detailed discussion of these terms, consider reading Carpenter et al. (2015).

Problem A—Multiplication (5 groups of 6)			
Did not solve	*Direct modeling*	*Counting*	*Fact recall—known fact*
Gavin	Dwayne	Niveah	Kyle
Jarod	Laila	Addison	
Taylor	Jade	Nicholas	
	Ada		
	Jack		
	Isabella		
	Cameron		
	Kyle		
	Amanda		

Problem B—Multiplication (8 groups of ten)			
Did not solve	*Direct modeling by ones*	*Direct modeling by tens*	*Direct place value*
Cameron	Ada	Laila	Jack
	Isabella	Addison	Kyle
		Jade	
		Niveah	
		Dwayne	
		Jared	
		Gavin	
		Nicholas	
		Kyle	
		Taylor	
		Amanda	

Problem C—Measurement Division (how many tens in 67)			
Did not solve	*Direct modeling by ones*	*Direct modeling by tens*	*Direct place value*
Jade		Niveah	Addison
Dwayne		Isabella	Jared
Amanda		Taylor	Nicholas
Laila			Kyle
Cameron			
Ada			
Jack			
Gavin			

Figure 1. Classification of students' strategies for each of the three problems

Students will understand the situation in a measurement division word problem.

Because so many students struggled with solving the measurement division problem, the teachers decided they would pose another measurement division problem and focus on providing support for students to comprehend the situation described in the problem. Several students struggled with both understanding the situation in the problem and working with the greater numbers. The teachers decided to change the problem used in the classroom lesson to involve smaller numbers and decided to remind students to use manipulatives or draw pictures if it helped them to think about the numbers in the problem. The students who had solved problem C successfully, and those who solved it successfully and quickly during the classroom lesson, would be given an opportunity to solve a similar problem involving numbers greater than 100.

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Planning for the Lesson

The team worked to develop the following measurement division problem.

Ms. Osborn has 32 Skittles. Each party bag holds 10 Skittles. How many party bags can Ms. Osborn fill?

Rationale for the Problem Selected

The teachers wanted to pose another measurement division problem so that those students who did not understand the situation could be given an opportunity to make sense of its scenario. They used the numbers 32 and 10 in hopes that using smaller numbers might help some students who perhaps understood the problem but struggled with higher numbers. They used Skittles in the problem, because the class liked those candies and had experience with them.

Strategy for Differentiation to Meet the Needs of All Students in the Class

The teachers prepared a second set of numbers, 132 and 10, for students who were ready for the challenge of numbers greater than 100. These numbers were chosen, because 132 is exactly 100 more than 32.

Notes on What to Notice about Student Thinking

While students solved the new problem, the teacher intended to make note of which students were using a strategy different from that they used before and planned to ask those students to share their thinking with the rest of the class at the end of the lesson.

Lesson Plan

In planning for this lesson, the teachers developed the following learning goal and associated lesson plan:

Students will understand the situation in a measurement division word problem.

1. Say, "You solved some problems earlier, and I saw several different strategies that were used to solve the problems."

2. Introduce the problem, saying, "I am going to give you another problem to solve."

3. Display the following problem.

   Ms. Osborn has ___ Skittles. Each party bag holds ___ Skittles. How many party bags can Ms. Osborn fill?

4. Read the problem aloud to the class. As you read, say "blank" in place of the number. This practice allows students to think about the situation in the problem without initially focusing on the numbers involved. Then reread the problem with students also reading aloud.

5. Read the first sentence of the problem. After reading the first sentence, ask the class, "What do we know so far about the problem?" "Who knows what Skittles are?"

6. After students respond, read aloud the second sentence of the problem. Ask the class, "What is Ms. Osborn going to do with the Skittles?"

7. Read aloud the last part of the problem. Ask, "What are we trying to find out in this problem?"

8. Then, write the numbers in the problem (32, 10).

9. Read the problem aloud with the numbers included.

10. Tell students that they are going to solve this problem and when everyone has finished, a few will come up to share their strategies.

11. Provide students with the problem either displayed or on a sheet of paper. Also, provide students with manipulatives and writing materials.

    Ms. Osborn has 32 Skittles. Each party bag holds 10 Skittles. How many party bags can Ms. Osborn fill?

12. Read the problem aloud as many times as necessary. Remind students that they may use whatever strategy makes the most sense to them.

13. Provide time for students to solve the problem.

14. Circulate around the room and identify those students who would be good candidates for sharing their strategies on this new problem.

15. While circulating, stop and ask any clarifying questions in order to be certain you understand students' strategies and thinking. Make note of any students using the following strategies:

    a. *Unable to solve*
    b. *Direct modeling by ones*
    c. *Direct modeling by tens*
    d. *Direct place value*

16. Examples of clarifying questions include:
    a. What does this set of objects represent?
    b. How did you count the objects? Can you count out loud for me?
    c. I see that you wrote three tens. What do those numbers represent?

17. Students who successfully solve the problem can be given the second set of numbers (132, 10) and asked to solve the problem with those numbers.

18. Identify three or four students to share their strategy with the class in the closing portion of the lesson. Consider selecting strategies that span the level at which most of the students are solving the problem. Consider having students share strategies from least sophisticated to most sophisticated. Prepare to encourage students to make connections among different strategies and help them express their ideas both verbally and in writing. If possible, let these students know they will be asked to share with the class during whole-group discussion.

19. Call students back to the whole group for a final discussion.

20. Say, "You just solved a problem, and I saw several different strategies that different students used to solve it."

    "We are going to look at the problem you solved and some of you will get to share your thinking so that we can hear more about how you were thinking of the problem. Think about how you solved this problem."

21. Draw students' attention to the written problem.

22. Use the following questions to structure a discussion while allowing students time to respond.
    a. "Think about how you solved that problem. Can you read the problem with me?" Read the problem aloud with students.
    b. "Picture 32 Skittles in your head. Each party bag holds 10 Skittles. Does that make sense? Do you have a picture in your mind?"
    c. "Now, listen carefully as students share their strategies. As you listen, think about how the strategies being shared are the same as or different from the strategy you used."

23. Invite students to share their strategies one at a time. As students share, listen for opportunities for the students to help the sharing student to communicate if he or she is struggling. Consider having students share strategies in the following order:
    a. Direct modeling by ones
    b. Direct modeling by tens
    c. Direct place value

Reflection

During the sharing portion of the lesson, five students who did not understand the situation in Problem C during the interview were able to model the second problem successfully and understand that some Skittles were left out of bags. One of the students who solved by direct modeling by tens understood a student who used *direct place value* to explain his thinking.

The teachers agreed that posing additional measurement division problems involving groups of 10 would be beneficial, so that more students could make the connection between how many tens are in a two-digit number and the leading digit in a two-digit number. This process may help students advance their understanding to the point where they can use a *direct place value* strategy with understanding.

What's Next?

Stories of teachers engaging in collaborative inquiry focused on using student thinking to inform instructional decisions

What's Next? is a collection of stories documenting professional development experiences shared by elementary teachers working collaboratively to study the complex process of teaching and learning mathematics. Each story in the collection describes practicing teachers studying the thinking processes of real students and using what they learn about those students to make decisions and try to help advance those students' understanding on that day.

The teachers in each story start by learning about how individual students are solving a set of mathematics problems. They use this freshly gathered knowledge of student thinking to develop near-term learning goals for students and a lesson plan tailored to specific students on that specific day. One of the teachers implements the planned lesson while the other teachers observe in real time. The teachers then gather to discuss and reflect on their observations and insights.

In these lessons, the practice of teaching is slowed way down. The stories tell of teachers who are studying student thinking and using that information to plan and implement instructional decisions at a pace that is much slower than it occurs in daily practice. The stories in this collection also depict many aspects in common with formative assessment and lesson study, both of which are a process and not an outcome.

The stories depict real situations that occurred in real time and include both successes and shortcomings. We hope that the stories may be studied and discussed by interested educators so that the lessons and ideas experiences of these teachers and instructional coaches may contribute to additional learning and sharing among other interested teachers.

Learn more about these and other stories at **http://www.teachingisproblemsolving.org/**