Making Connections Between Vincent's Model and Matthias' Equations to Improve Understanding of Addition With Large Numbers

This story is a part of the series:

***What's Next?* Stories of Teachers Engaging in Collaborative Inquiry Focused on Using Student Thinking to Inform Instructional Decisions**

What's Next?

Stories of teachers engaging in collaborative inquiry focused on using student thinking to inform instructional decisions

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Introduction

This lesson focuses on first-grade students' growing understanding of the base-ten number system. Students share a variety of strategies to add two two-digit numbers. Connections are drawn between models that differentiate between tens and ones and their numerical representations.

Relevant Florida Mathematics Standards

MAFS.1.NBT.2.2 Understand that the two digits of a two-digit number represent amounts of tens and ones.

a. 10 can be thought of as a bundle of ten ones — called a "ten."

b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.

c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).

d. Decompose two-digit numbers in multiple ways (e.g., 64 can be decomposed into 6 tens and 4 ones or into 5 tens and 14 ones).

Background Information

Consider reading chapter seven in *Children's Mathematics: Cognitively Guided Instruction* (Carpenter et al., 2015). This chapter provides background on the varied ways children solve addition and subtraction problems with numbers greater than 20. It also expands upon the strategies explained in the *Analyzing Student Thinking* section.

Carpenter, T. P., Fennema, E., Franke, M. L., Levi, L., & Empson, S. B. (2015). *Children's Mathematics: Cognitively Guided Instruction (2nd. Ed.).* Portsmouth, NH: Heinemann.

Analyzing Student Thinking

A group of teachers explored individual first graders' understanding of tens and ones in the context of professional development. Each of these interviews lasted approximately 30 minutes. Each student was asked to solve four problems in one-on-one interviews with a teacher. The teachers read the problems out loud—one at a time—and asked the students to solve them in whatever way they thought best. Teachers were instructed to pay close attention to students while they were solving each problem so as to understand the students' thinking and to take detailed notes.

Problem A. A basket had 20 apples in it. You put 10 more apples in the basket. How many apples are in the basket now?

Problem B. Maria has 30 shells. How many more shells would she need to find on the beach to have 55 shells?
Alternate numbers: (30, 40)

Problem C. Carlos had 27 books. He got 23 more books for his birthday. How many books does Carlos have now?
Alternate numbers: (12, 25)

Problem D. Rob had 50 rocks. He lost 22 of them. How many rocks does Rob have now?
Alternate numbers: (30, 12)

Almost all the students were expected to solve the four planned problems with the numbers in the text. Several other accommodations were created to enable teachers to adjust the interview to other individual students' ability levels. To accommodate students who could not solve the problems as written or had limited knowledge of multidigit numbers, a carefully selected pair of alternate numbers were provided for problems B through D. If teachers perceived the students to be feeling frustrated or overwhelmed by the first interview problem, they were instructed to pose the following problem in its place.

Alternate Problem A2. Let's pretend that you had 20 jellybeans. If you ate 9 of those jelly beans, how many jellybeans would you have left?

Depending how students in this latter group solved the alternate problem, the teachers could next choose to try the interview again with numbers in the range of the jellybean problem or with single-digit numbers. If the child was really distressed, the interviewing teacher could take a break, start chatting about something else before returning to the interview, or abandon the interview altogether.

The students had access to linking cubes, base-ten materials, and paper and pencil for all problems. As students responded to each problem, the teachers took notes about the students' thinking processes and prompted for more details whenever they considered it necessary. If a student solved a problem mentally, with cubes, or on their fingers, the interviewing teacher asked the student to show that process on paper. The teachers were aware that what the students wrote in these instances might be only a partial explanation of the thinking process. They nonetheless considered this information to be valuable to include with teachers notes for examination of the students' proficiency with mathematical notation.

After the interviews, the teachers shared their observations about the first graders' thinking with each other. They chose the Carlos's books problem (Problem C) as the basis for discussion, because it had a straightforward semantic structure and involved numbers that allowed for a variety of strategies. The teachers categorized the students on the basis of the strategies they used on the problem. A summary of the working definitions of the student strategy categories is given below.[1]

Multidigit Computation Strategies

A student who uses a *direct modeling with ones* strategy represents each multidigit number in the problem as a set of ones using manipulatives or pictures to represent each quantity in the problem and then counts the objects or pictures to determine the answer. For example, a student using this strategy for the Carlos's books problem might first create a set of 27 objects, then create another set of 23 objects, and finally count the objects in the full set by ones.

A student who uses a *counting by ones* strategy does not physically represent each quantity in the problem. Fingers, objects, or tally marks are often used to keep track of the number of counts. For the Carlos's books problem, for example, a student using a *counting by ones* strategy might say "27" and proceed to count forward by ones 23 times.

A student who uses a *direct modeling with tens* strategy represents each multidigit number in the problem using manipulatives or pictures that reflect the base-ten structure of the number system (e.g., with base-ten blocks or base-ten pictures). Then, the student counts the objects or pictures by tens and ones to determine the answer. In Carlos's books problem, the student might use base-ten blocks to model both 27 and 23 by representing 27 with two ten rods and seven unit cubes then model 23 with two ten rods and three unit cubes. The student might then count the number of objects by tens and ones. For example, the student might say, "10, 20, 30, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50." (The student might also replace the ten ones with a ten, count the ten rods by ones and consider five ten rods to be 50.) If the student counted from one to 50 by ones, the student's strategy would still be considered *direct modeling with tens* (albeit, an inconsistent use of tens), because the student created a representation involving groups of tens and ones.

A student who uses an *invented algorithms* strategy demonstrates flexibility in thinking about numbers by decomposing them and recomposing them in *ad hoc* ways without the use of ma-

1 The descriptions of strategies presented here are the current descriptions used by our team, and we consider them to be fluid, as our understanding of these ideas continues to evolve. For a more detailed discussion of these terms, consider reading Carpenter et al. (2015).

Direct modeling with ones	Counting by ones	Direct modeling with tens		Invented algorithms	Other
		Inconsistent use of tens	Consistent use of tens		
Jennifer	Pablo	Vincent	Sarah	Matthias	Herick
Ryhanna	Sarah	Ruby	Lara	David	Kaitlyn
			Antoine		Sophia
			Allessandra		Eric
			Ashton		
			Zane		

Figure 1. Classification of students based on strategies used.

nipulatives or pictorial models. A student using an invented algorithm strategy might operate on the tens and ones separately and then combine the partial sums to get a final result. For example, a student solving the books problem might think about adding the two tens from 27 and the two tens from 23 to make 40 and about adding the seven and three to make a ten. The student might then say, "40 + 10 is 50." Another possible invented algorithm strategy would be to increase or decrease partial sums or differences. For the same problem, a student might say, "27 and three is 30, 30 and 20 is 50."

A student might also attempt a strategy that does not clearly fit into one of the strategies listed above, and the teachers in this group decided to classify these as *other*.

Strategies used by students in this classroom

Figure 1 shows how the students were classified on the basis of the strategies they used for the Carlos's books problem. The most common category of student thinking observed in the interviews for this problem was the *direct modeling with tens* strategy.

During the discussion, the teachers observed that students were beginning to understand that ten ones make one ten. Some of the students showed a more consistent understanding of the concept than others. Pablo and Sarah counted the first number by tens and ones but counted on the second number by ones. Ruby also showed some understanding of grouping by tens by modeling the two numbers with tens and ones and counting the first number as, "10, 20, 21, 22, 23, 24, 25, 26, 27," but she continued counting the two tens in the second number as ones, saying, "28, 29, 30" and gave the answer as 30.

The teachers found Vincent's strategy interesting, because it illustrated a direct modeling strategy transitioning from modeling with ones to modeling with tens. Figure 2 shows Vincent's work. He modeled each object individually with circles but separated the circles into groups of ten by vertical lines. When asked what those lines meant, Vincent answered, "That's a ten." He nonetheless found the answer of 50 by counting each circle by ones.

All students who used a *direct modeling with tens* strategy solved the Carlos's books problem in similar ways. Zane, for example, modeled 23 in a picture by drawing two tens and three ones

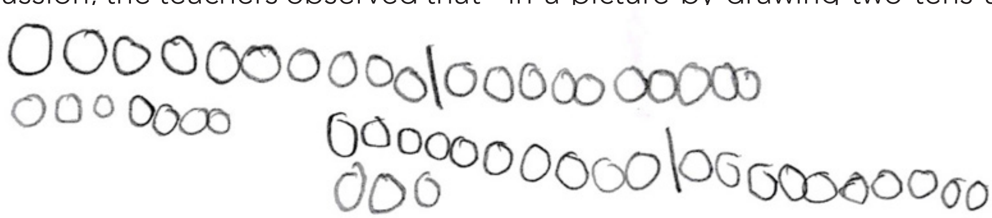

Figure 2. Vincent's drawing of direct modeling with ones *grouped by tens*

and modeled 27 by drawing two tens and seven ones. Then, he redrew the four tens together and the ten ones together and counted them saying, "10, 20, 30, 40, 41, 42, 43, […], 50". Mathias, who solved the problem using an invented algorithm strategy, found the answer by a strategy similar to Zane's but did not use a model for the numbers. He explained that the two and two make four, the seven and three make ten, and that together makes 50. When asked how that makes 50, Mathias elaborated on his answer by saying that the twos were in fact 20 each, so the four was in fact 40.

The teachers found Vincent's strategy interesting, because it illustrated a direct modeling strategy transitioning from modeling with ones to modeling with tens.

The teachers' overall conclusion after examining the strategies used in all interview problems was that the problem type and numbers used greatly influence the strategies the children used. They noticed that students were more likely to use safer, less complex strategies—such as *direct modeling with ones*—to solve the change-unknown problem and were more likely to use the more advanced strategies to solve the result-unknown problems. They also noticed that several students made notation errors when trying to write mathematical expressions that matched their strategies. On the basis of these observations, the teachers decided to develop a lesson centered around the following two learning goals:

1. *Students will make connections among different strategies that use ten as a unit.*

2. *Students will improve their mathematical notation.*

The teachers agreed that it might be a good time for students who were *direct modeling with tens* to hear from students who used an invented algorithm strategy. They also thought the students who used *direct modeling with ones* strategies might benefit from hearing other students who counted by tens and ones. Finally, those students who struggled with the verbal counting sequence when *counting by ones* might benefit from hearing and helping other students count aloud.

Planning for the Lesson

The teachers worked to develop the following word problem:

> *26 boys threw pies at Awesome Austin. 34 girls threw pies at Awesome Austin. How many kids threw pies?*

Rationale for the selected problem

The context of the story in the problem was familiar to the students, because they participated in the school's Fun Run on the previous day. During the Fun Run, they witnessed a fundraising segment in which a comedic character—Awesome Austin—had pies thrown at him by the audience for a small donation. The teachers thought this context would motivate the children and introduce some fun into the problem.

The numbers 26 and 34 were selected to encourage a variety of strategies involving various groupings of tens and ones, like those the teachers observed students using to solve the Carlos's books problem.

The teachers chose to use a *part-part-whole, whole unknown* problem type to work toward the chosen learning goal. (For a description of problem types, see chapter two in Carpenter et al., 2015.) This is one of the easiest types of addition word problems for students to understand and solve correctly. They decided to use this simple problem type to reduce the cognitive demands that come with making sense of the story problem so that students could focus on understanding new strategies for adding numbers and on learning about operations on multidigit numbers with understanding.

After analyzing the distribution of strategies in Figure 1, the teachers decided to ask one student from each strategy group to share his or her strategy on the Awesome Austin problem. Teachers thought that most students would use strategies on the Awesome Austin problem similar to those they used on the books problem, because the problems were very similar. The plan was that, as students shared their strategies, an effort would be made to connect the ideas presented in the various strategies across the progression of thinking.

Strategy for differentiation to meet the needs of all students in the class

This lesson was organized to allow the students sufficient time to work out the answer for themselves before having the whole-group discussion. To ensure that all learners had entry points for the problem, every student was provided with physical manipulatives such as linking cubes and counters. They also had access to paper and pencils.

Students would sit in a circle on the carpet at both the outset of the lesson—when the problem was introduced—and for the last part of the lesson, during the whole class discussion about the strategies used. This arrangement would limit the distractions students might have at their desks and would engage students more with the teacher and with each other. How students would be arranged on the carpet during the discussion time was also considered to be important. The teachers decided to seat students who used similar strategies next to each other, for two main reasons. First, this arrangement would place the students who used strategies different from that being explained in a position to face the talking student and more easily see how that student was using the manipulatives. Second, having students with similar strategies sitting next to one another might encourage them to correct each other and to help each other grasp new ideas, because all these students would be at a similar level of understanding of numbers.

The teachers planned to start with the simplest strategy—*direct modeling with ones*—to provide a proactive opportunity to include the students who were not yet using grouping-by-tens strategies in the discussion.

The teachers decided to use two-digit numbers. The numbers were carefully chosen so that students *counting by ones* would not find the process too laborious but that the greater efficiency of use strategies involving groups of ten would be apparent.

The teachers though it important, while making connections between strategies during the whole-class discussion, also to teach students how to use written notation effectively. This skill was considered particularly important for the students who were using cubes or fingers to solve the problems. The teachers conjectured that learning to translate their drawings into mathematical equations might help students internalize the mathematics and also to understand a more complex strategy.

The focus during the work time was for the teacher to observe the range of different strategies used, particularly to find students who used the concept of tens in any way. The teacher paid particular attention to students who were categorized as using other strategies in Figure 1. If these students had good strategies for solving the problem or some interesting thinking, the teacher wanted to highlight them during the whole-class discussion to increase their confidence in their own work.

Lesson Plan

The following lesson took place in a first-grade classroom. It lasted approximately 40 minutes and was focused on the problem the teachers developed in the Planning for the Lesson segment, which was developed to address the following learning goals:

> *Students will make connections among different strategies that use ten as a unit.*
>
> *Students will improve their mathematical notation.*

1. Sit on the carpet in front of the classroom and ask the first graders to gather around the perimeter of the carpet.

2. Post the Awesome Austin problem for the students to see: 26 boys threw pies at Awesome Austin. 34 girls threw pies at Awesome Austin. How many kids threw pies?

3. Introduce the problem, saying, "I have a story for you to figure out, and it has to do with the Fun Run you participated in yesterday." Read the problem once, then read it again a second time.

4. Ask questions to make sure the students understand what happened in the Awesome Austin problem.

   a. Ask the students, "Who is Awesome Austin? Why are people throwing pies at Awesome Austin's face?"

   b. Ask, "How many boys threw pies at Awesome Austin?"

   c. Ask, "How many girls threw pies at Awesome Austin?"

   d. Say, "The story says the boys threw pies, and the girls threw pies. Who threw the most pies?"

   e. Ask, "What is the problem asking us to figure out?"

5. After discussing each of these questions, direct the students to go back to their seats and to solve the problem on their own. Be sure each student has access to physical manipulatives and paper and pencils.

6. As the students work on the problem, walk around and study how students solve it. If individual students' strategies are not clear, ask them quietly to explain their thinking. If a student solved the problem using linking cubes or fingers, ask the student to write the numbers used on paper.

   a. Take notes on the strategies the students used.

   b. Make the final choice of students who will be asked to share their strategies with their peers and the order in which they will be asked to share.

c. Be sure to tell those selected students that they will be asked to explain their thinking to their classmates and to bring their tools with them to the carpet. These tools might be their papers with the written strategies or the physical manipulatives they used to solve the problem.

The remainder of this lesson plan tells the story of how the rest of the lesson unfolded in this particular classroom.

7. When all or almost all students had finished the problem, the teacher asked students to return to the carpet in the front of the classroom.

   a. She asked all students except those who were going to be sharing to leave their cubes, pencils, and papers at their desks.

   b. She arranged the students so that those who used similar strategies on the interview problems (as shown in Figure 1) sat next to each other.

8. The teacher said, "I'm so excited to see how you solved this problem." She asked Sophia to come to the chart stand and draw on paper what she did. Figure 3 shows the chart paper with all the strategies discussed during this lesson, including Sophia's. Sophia's strategy was written in pink.

   a. While Sophia performed this task, the teacher read the problem aloud again.

   b. Sophia drew 26 circles in a row and then drew 34 other circles below those, in a second.

   c. The teacher asked the other students whether they knew what Sophia did. The majority of students said they understood Sophia's picture.

   d. Sophia explained that she drew the two

They noticed that students were more likely to use safer, less complex strategies—such as direct modeling with ones—to solve the change-unknown problem and were more likely to use the more advanced strategies to solve the result-unknown problems.

numbers out with circles then counted them together.

e. The teacher asked Sophia to count the circles out loud. She counted 60 circles, one by one.

f. The teacher said to Sophia, "On you paper, you wrote that you counted 63, that's different from the number of circles you just counted. Why is that?"

g. Sophia appears hesitant to answer. She counted the circles on the chart again and said, "There are 60 circles. This is the right answer."

h. The teacher asked the other students, "What do you think could have happened?"

i. One student offered his opinion that some of the circles are really close together.

j. The teacher agreed, "It's hard sometimes to count so many circles."

The teachers conjectured that learning to translate their drawings into mathematical equations might help students internalize the mathematics and also to understand a more complex strategy.

9. The teacher next asked whether any students used cubes to solve this problem. Vincent raised his hand, and the teacher asked him to show his peers how he used cubes to solve the problem.

a. Vincent had linking cubes with him. The linking cubes were on the carpet, and the students were seated in a circle so they could see the cubes. Vincent separated his linking cubes into a group of 26 cubes and a group of 34 cubes. The first group consisted of two rods of ten linked cubes each and six other cubes linked together. The second group consisted of three rods of ten cubes each and another four cubes linked together. He then pointed to the smaller group and said, "This is the 24 boys who threw pies."

b. The teacher asked, "How do you know this is 24 boys?"

c. Vincent answered, "There are two tens (pointing at the two long rods) and one that's shorter (pointing at the six connected cubes)…"

d. The teacher interrupted Vincent's explanation, covered the six connected cubes with her hand and then asked the class, "How many are in the shorter stick?" Some were unsure of the answer. Others were confident they had the answer and raised their hands eagerly. One student answered, "Six."

e. The teacher next pointed to the larger pile of cubes and asked Vincent, " How many do you have there? How do you know?"

f. Vincent moved each rod a bit to the left and counted as he did so, "10, 20, 30, 34."

g. The teacher asked, "How many are in the short stick?" Vincent answered, "Four."

h. The teacher urged him to continue. Vincent then brought all the rods together. He arranged the long rods in a line next to each other, then connected the shorter rod of six with the other rod of four cubes and placed the newly formed long rod next to the others. Vincent then counted, pointing at each rod, "10, 20, 30, 40, 50, 60."

i. The teacher asked class, "How did Vince count the cubes?" She chose Ashton from the set of students who raised their hands and asked him to count the cubes as Vincent did. The teacher gave Ashton Vincent's cubes, and he used the cubes to answer.

j. Ashton made the same moves with the cube that Vincent had—albeit in a different order—and said, "He put those four and six cubes together and that made a ten. And he counted the tens, 10, 20, 30, 40, 50, 60."

10. The teacher thanked the two boys and continued speaking with the whole group, "I know you usually write your problems in your math journal. How can you show Vincent's work in the math journal?"

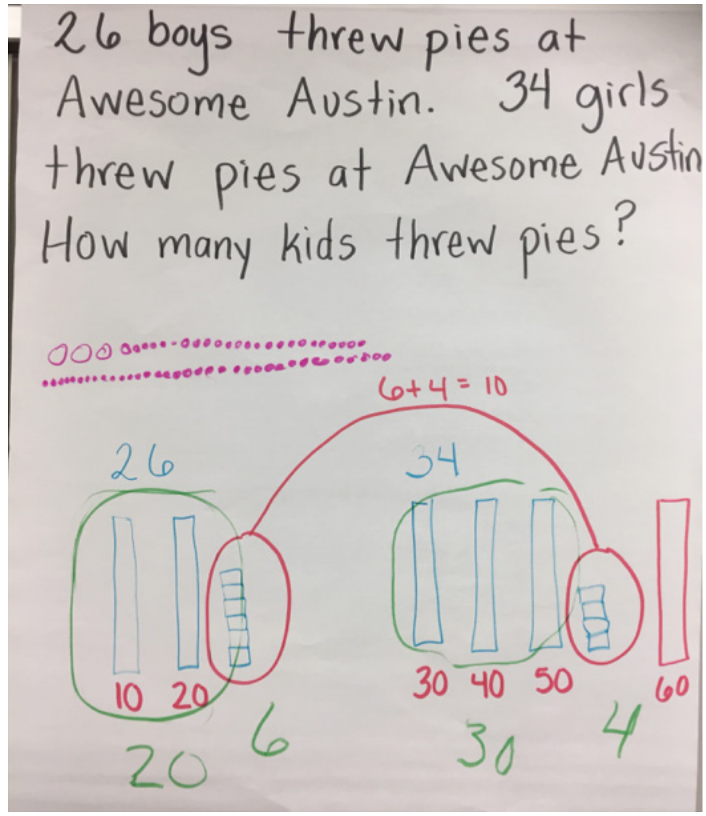

Figure 3. Depictions of students' strategies explained during the class discussion for the Awesome Austin problem

a. When no one answered her, the teacher wrote 26 on the chart pad. She asked the class, "How did Vincent make 26?"

b. A student answered, "He got two ten sticks and a six."

c. The teacher drew two vertical rods, as seen on the bottom left half of Figure 3, in blue marker. She asked, "Does this look like the tens sticks?" The class answered in the affirmative.

d. The teacher next drew six connected squares next to the two vertical rods.

e. She wrote the number 34 on the chart and asked the first graders, "How many tens sticks do we need to show 34?"

f. Another student answered, "Three sticks and then four more."

g. The teacher drew three vertical rods and four connected squares under the number 34."

h. She then asked, "And then what did Vincent count first?"

i. The students answered in chorus, "The tens!"

The connections drawn between Matthias' *strategy and Vincent's model may have been the most productive part of the lesson. The students appeared highly engaged and attentive during this part, even though they were at different levels of understanding of the numerical connections. More such connections would be needed in subsequent lessons to cement those emerging ideas about numbers.*

j. The teacher asked the students to count with her as she wrote the partial sum under each rod. She wrote 10, 20 in red under the two rods illustrating 26 and wrote 30, 40, 50 under the three rods showing 34 (see Figure 3).

k. The teacher continued, "So, what did he do next?"

l. A student answered, "He put the six and four together."

m. The teacher pointed to the six and the four squares and asked, "You mean these? And how much is that?"

n. The student said, "ten." The teacher circled the two groups of squares, connected them with an arc, and wrote on the chart 6 + 4 = 10 in red. She then drew a sixth rod in the illustration and wrote 60 underneath.

o. The teacher said, "So, that's how he got 60 by combining the leftover cubes."

11. The teacher continued, "We're going to see another strategy. Matthias, can you please explain what you did?"

a. Matthias explained his written strategy verbally, "I did two plus three equals five, so that means 20 plus 30 equals 50. And I did four plus six equals 10. So, 50 plus 10 equals 60."

b. The teacher wrote on another chart paper 26 + 34 = __, explaining, "This is a story about 26 boys and 34 girls". She then asked Matthias to write all the equations he just discussed on the chart paper as well. Matthias wrote, 2 + 3 = 5, so 20 + 30 = 50//6 + 4 = 10//50 + 10 = 60

c. The teacher said, " I see you wrote here, 20 plus 30 is 50. Where is that in our story? Where is the 20 + 30 in Vincent's strategy?"

d. Several students were eager to answer. The teacher chose a student who got up and pointed at the two rods labeled 10 and 20 (Figure 3).

e. The teacher circled the two rods the student pointed at with a green marker (Figure 3) and wrote 20 underneath. She then said, "Does everyone see the 20? Where's the 30?"

f. Another students came to the chart and pointed to the three rods labeled 30, 40, 50.

g. The teacher circled those three rods and wrote 30 underneath. She continued, "Where is 6 + 4 in Vincent's pictures?"

h. Another student pointed to the six and the four cubes and the teacher wrote 6 and 4 in green underneath.

12. The teacher next said, "I have a question for you. Is 26 plus 34 the same as doing this?" She wrote 26 + 34 = 20 + 30 + 6 + 4 on the second chart paper. "Is this true? Tell a partner."

13. The students chose a shoulder partner and started discussing whether the equation was correct. The teacher allowed them a couple of minutes to discuss their thinking with each other. She joined a couple of the pairs to listen in on the student's opinions and to ask them questions about their thinking.

14. The teacher concluded the lesson by thanking everyone for using their brains so much.

Reflection

What we learned from the lesson

The connections that were made between the *direct modeling with tens* strategy and the *invented algorithm* strategy seemed interesting to the students. Even though they were not accustomed to this type of lesson, they were able to adapt quickly to new lines of questioning and explaining their thinking. With some initial prompting, the students were able to pick up on the similarities and differences between the different strategies. The context of the problem was interesting enough for the children to become engaged in the story as well. They were excited to solve a mathematical problem about a real-life situation they experienced and enjoyed.

Vincent, the student chosen to share his *direct modeling with tens* strategy, was purposefully selected, because he demonstrated an inconsistent use of tens during the initial interview. The teacher noticed his correct use of tens with manipulatives while solving the Awesome Austin problem. When asked to record on his paper what he did, however, Vincent was not able to model his strategy accurately or to write a matching mathematical expression. The teacher used the opportunity to highlight his thinking to himself and his peers while modeling for him how to record his strategy. This move was considered effective by the teachers, because it came across as an instructional segment for the whole class and not as a correction for Vincent.

Sarah's strategy was thought to be important to address in the whole class discussion for a variety of reasons. It acknowledged the students who did not yet grasp the concept of grouping by tens and positioned the *direct modeling with ones* as a valid strategy for solving the problem. It also emphasized the difficulty of counting so many single units accurately. Sarah had initially miscounted her circles during individual work. The teacher took that opportunity to make all the first graders aware of the difficulty of counting large quantities by ones. Although exposing Sarah's mistake to her peers might have become embarrassing for her, this possibility was turned into a positive, because her later counting was correct, and she was able to figure out what went wrong. Looking back, the teachers did acknowledge that more time spent highlighting the similarities and differences between Sarah's and Vincent's models might have helped the students who did not yet grasp grouping by tens more than just sharing them side by side.

Matthias' strategy was a good way to engage the students with more advanced knowledge in the discussion. His work was particularly useful, because he was clear and accurate in his notation. Even though he initially referred to the tens in the numbers as ones, he clarified what he meant immediately. The connections drawn between Matthias' strategy and Vincent's model may have been the most productive part of the lesson. The students appeared highly engaged and attentive during this part, even though they were at different levels of understanding of the numerical connections. More such connections would be needed in subsequent lessons to cement those emerging ideas about numbers. In addition, this discussion provided opportunities to teach mathematical notation and how to do translate models into mathematical expressions.

The final task given to the students was deliberately not addressed afterwards by the teacher . She considered that much more experience focusing on the meaning of the equals sign was needed for a productive discussion. This opportunity was nonetheless seized to have the first graders start noticing new ways of using the equals sign and be thinking about what it means.

Next steps based on the outcome of the lesson

This lesson was successful, overall, in focusing on understanding of multidigit numbers as groups of tens and ones. A follow-up lesson might involve a problem adding two numbers whose ones make more than ten, such as 27 and 35. This problem would be more difficult than the problems included here and would further challenge them to think about grouping tens and ones. In addition, the students seemed to learn from one another and might benefit from more opportunities to share their strategies with the class in the future. Further opportunities to encourage certain types of *invented algorithms* can be achieved by a careful choice of the problem numbers.

What's Next?

Stories of teachers engaging in collaborative inquiry focused on using student thinking to inform instructional decisions

What's Next? is a collection of stories documenting professional development experiences shared by elementary teachers working collaboratively to study the complex process of teaching and learning mathematics. Each story in the collection describes practicing teachers studying the thinking processes of real students and using what they learn about those students to make decisions and try to help advance those students' understanding on that day.

The teachers in each story start by learning about how individual students are solving a set of mathematics problems. They use this freshly gathered knowledge of student thinking to develop near-term learning goals for students and a lesson plan tailored to specific students on that specific day. One of the teachers implements the planned lesson while the other teachers observe in real time. The teachers then gather to discuss and reflect on their observations and insights.

In these lessons, the practice of teaching is slowed way down. The stories tell of teachers who are studying student thinking and using that information to plan and implement instructional decisions at a pace that is much slower than it occurs in daily practice. The stories in this collection also depict many aspects in common with formative assessment and lesson study, both of which are a process and not an outcome.

The stories depict real situations that occurred in real time and include both successes and shortcomings. We hope that the stories may be studied and discussed by interested educators so that the lessons and ideas experiences of these teachers and instructional coaches may contribute to additional learning and sharing among other interested teachers.

Learn more about these and other stories at **http://www.teachingisproblemsolving.org/**