



Jeremiah Makes Sense of Tens and Ones

This story is a part of the series:

***What's Next? Stories of Teachers Engaging in Collaborative Inquiry Focused on
Using Student Thinking to Inform Instructional Decisions***

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What's Next?

Stories of teachers engaging in collaborative inquiry focused on using student thinking to inform instructional decisions

Editors

Robert C. Schoen
Zachary Champagne

Contributing Authors

Amanda Tazaz
Charity Bauduin
Claire Riddell
Naomi Iuhasz-Velez
Robert C. Schoen
Tanya Blais
Wendy Bray
Zachary Champagne

Copy Editor

Anne B. Thistle

Layout and Design

Casey Yu

Workshop Leaders

Linda Levi (Coordinator)
Annie Keith
Debbie Gates
Debbie Plowman Junk
Jae Baek
Joan Case
Luz Maldonado
Olof Steinhorsdottir
Susan Gehn
Tanya Vik Blais

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Introduction

Based on a day spent with a second-grade class at the beginning of the school year, this story describes how multiplication and division word problems involving groups of ten were used to assess and advance students' understanding of place value in the base-ten number system. The resulting lesson, developed collaboratively by teachers in the context of professional development for those students in that class on that same day, illustrates how focus on a story-problem situation and students' varied ways of approaching the problem can be used to increase students' understanding that ten can be thought of simultaneously as ten ones and one ten.

Relevant Florida Mathematics Standards¹

MAFS.1.NBT.2.2 Understand that the two digits of a two-digit number represent amounts of tens and ones.

- a. 10 can be thought of as a bundle of ten ones — called a “ten.”
- b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.
- c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).
- d. Decompose two-digit numbers in multiple ways (e.g., 64 can be decomposed into 6 tens and 4 ones or into 5 tens and 14 ones).

Background Information

Chapters four and six from *Children's Mathematics: Cognitively Guided Instruction* (Carpenter et al., 2015) offer useful background information for the types of problems used in this lesson. Chapter four provides background on word-problem types that represent basic multiplication and divi-

sion and the trajectory of strategies early-grades learners most typically use to solve them. Chapter six explores how multiplication and division word problems involving grouping by tens can be used to develop early-grades learners' understanding of the base-ten number system.

Carpenter, T. P., Fennema, E., Franke, M. L., Levi, L., & Empson, S. B. (2015). *Children's Mathematics: Cognitively Guided Instruction* (2nd. Ed.). Portsmouth, NH: Heinemann.

Analyzing Student Thinking

In the context of a professional-development experience, a group of teachers conducted brief individual interviews with students in a second-grade class for which they would later be developing and implementing a lesson. The purpose of the interviews was to learn how the second-grade students were using knowledge of place-value concepts in their strategies for approaching word problems involving groups of ten.

In an interview lasting 10–15 minutes, each second-grade student was asked to solve the following word problems:

Problem A. I have 8 boxes of pencils with 10 pencils in each box. How many pencils do I have?

Problem B. The second graders at Darnaby Elementary School raised \$67 to buy books for the children's hospital. If each book costs \$10, how many books can they buy?

As each problem was posed, students were encouraged to pay close attention to the problem situation and to generate their own way of solving the problem. Students had access to tools including base-ten blocks, linking cubes, paper, and pencils. The students were not required to use any specific tool; they were told explicitly that they could use the tools provided, use their fingers, or solve the problems mentally. Interviewers

¹ The lesson described took place in a second-grade class at the beginning of the school year. On the basis of what teachers learned about the second-grade students' understanding of place value through one-on-one interviews, a classroom lesson was designed that was most closely aligned with a foundational first-grade standard.

attempted to support students' understanding of the problem, as needed, and asked probing questions to improve their understanding of students' mathematical thinking, but interviewers were careful to avoid teaching or showing students how to solve the problems.

Types of Strategies Students Commonly Use to Perform Multidigit Computation Problems²

A student using the *counts by ones* strategy represents all the quantities in the problem (grouping ten in each group) and counts by ones to determine the solution. For example, when asked to find the total number of pencils in 8 boxes of 10 pencils, a student might draw 8 rectangles with 10 dots in each and then count the dots one by one. For the problem involving finding the number of \$10 books that can be purchased with \$67, a student employing a *counts by ones* strategy might count out 67 snap cubes (by ones), partition the 67 cubes into groups of 10, and find the answer by counting the number of groups of 10 that have been made.

A student using the *counts by tens* strategy represents all the quantities and counts by tens and ones, keeping track with manipulatives or drawings (including fingers). For example, to find the total number of pencils in 8 boxes of 10 pencils, a student might use 8 base-ten rods to represent the 8 boxes of 10 pencils. Then the student would count the base-ten rods by tens, "10, 20, 30, 40, 50, 60, 70, 80," and recognize the last count (80) as the answer. For the problem involving finding the number of \$10 books that can be purchased with \$67, a student might count to 60 by tens, "10, 20, 30, 40, 50, 60," extending a finger for each count. Then the student would count or observe that 6 fingers are extended to determine the answer.

A student using the *direct place value* strategy knows how many tens and ones are in the given number and provides the answer without model-

ing. For the problem involving finding the total number of pencils in 8 boxes of 10, students employing *direct place value* would quickly answer 80. They would be able to justify that they know it is 80, because eight tens is 80. Similarly, on the problem involving finding the number of \$10 books that can be purchased with \$67, a student would quickly answer six and be able to explain that 60 dollars consists of six \$10's.

After the interviews, students' strategies for solving each problem were classified into broad strategy categories along a continuum of base-ten understanding: *counts by ones*, *counts by tens*, and *direct place value*. A fourth category, *did not solve with a valid strategy*, was created for students who did not solve or interpret the problem correctly or did not attempt to solve it.

Strategies Used by Students in This Class

The names of students in the class were written into a table (see Figure 1) for the teachers to view and analyze. After classifying the students by strategies for the two interview problems, the teachers discussed additional observations from the interviews. The teachers noticed that the majority of students employed a counts by ten strategy to solve the problems but that notable variation was apparent in the students' use of ten. For example, in solving the problem involving finding the number of \$10 books that can be purchased with \$67, some students knew immediately to represent the 60 with 6 tens with base-ten rods or sticks of connected snap cubes, whereas other students counted by tens (e.g., 10, 20, 30, 40, 50, 60) while pulling out a ten rod for each count to build the 60. The former strategy was considered to imply a more sophisticated place value understanding than that of students who needed to count by tens. A second observation was that some students classified as using a *counts by tens* strategy were inconsistent in their use of that strategy, sometimes reverting to counting by ones. In addition, multiple students observed using the counts by ten strategy struggled to determine the answer on the measurement division problem (i.e., finding the number of \$10 books that can be purchased with \$67). Some of them,

² The descriptions of strategies presented here are the current descriptions used by our team, and we consider them fluid, as our understanding of these ideas continues to evolve. For a more detailed discussion of these terms, consider reading Carpenter et al. (2015).

for example, built a model of 67 using a tens and ones structure, counted the tens, “10, 20, 30, 40, 50, 60,” and initially reported the answer as 60. With interviewer questions that refocused student attention on the problem details, these students were able to revise their answers to the 6, but they did not arrive at this answer without support from the interviewer.

Further more than one-fourth of the students were classified in the category did not solve with valid strategy. The most common approach used by students classified in this way was either to add the numbers in the problem together or, in the case of the second problem, to evaluate $67 - 10$. The teachers agreed that these students as well as some who used valid strategies would benefit from discussion that might aid them in making sense of the problem context in relation to students’ strategies, particularly a problem similar to Problem B. Through such a discussion, the teachers agreed that having the second-grade students explicitly examine the relationship between tens and ones would be helpful. For example, in Problem B, the teachers wanted the students to be able to identify the representations of tens as representing both \$10 and one book. They conjectured that focus on the different units (i.e., dollars, books) in the problem would help students

to coordinate their understanding of ten ones as being the same as one ten.

The following learning goals were established for the classroom lesson:

Students will understand how the problem context of a measurement division problem involving groups of ten is related to different student strategies.

Students will understand the relationship between tens and ones in a two-digit number, specifically that ten ones are simultaneously one ten.

Planning for the Lesson

In designing a lesson focused on the established learning goals for these students on this day, the teachers first spent considerable time deliberating over the details of the word problem that would be the focus of the lesson, especially the problem context and numbers. They agreed that the problem context should be familiar and interesting to students. They also wanted to make the units in the word problem discrete and easy to represent. In addition, the teachers decided

Problem type/problem	Strategies			
	Counts by ones	Counts by tens	Direct place value	Did not solve with a valid strategy
Multiplication I have 8 boxes with 10 pencils in each box. How many pencils do I have?	RJ	Royal	Maxwell	Darryl
	Samantha	Isaac	Wakeem	Arianna
		Kamryn	Jonathon	Jeremiah
		Osanna		Adrian
		Anthony		Vincent
		Keily		
		Linda		
	Leilani			
Measurement division The second graders at Maplewood School raised \$67 to buy books for the children’s hospital. If each book costs \$10, how many books can they buy?	RJ	Royal	Kamryn	Darryl
	Osanna	Leilani	Linda	Arianna
		Isaac	Jonathon	Keily
		Wakeem		Jeremiah
		Maxwell		Adrian
		Anthony		Samantha
		Vincent		

Figure 1. Classification of students on the basis of their strategies for solving the two word problems involving groups of ten.

With regard to numbers, the teachers conjectured that more students might interpret the problem correctly if the total number of items to be put in sets of ten were less than the 67 used in the interview problem.

to make the problem about one of the students in the class to encourage further personalization and engagement with the problem. With regard to numbers, the teachers conjectured that more students might interpret the problem correctly if the total number of items to be put in sets of ten were less than the 67 used in the interview problem. A few students appeared to struggle with their counting beyond 40. With these considerations in mind, the teachers decided to build a lesson for the second-grade students around having them solve and discuss the following measurement division problem:

Royal has 34 Pokémon cards. He wants to give 10 cards to every friend. How many friends can he share with?

Next, the teachers anticipated the varied strategies students might use to approach the Royal's Pokémon cards problem and discussed how they might select and sequence particular strategies to meet their established goals during a class discussion of students' solutions. The strategies anticipated by the group are presented in Figure 2.

The teachers agreed a desirable way to open was by having a student share a counting by ones strategy. The teachers reasoned that *counts by ones* is the most basic strategy and offered an entry point to understanding the word-problem context. The teachers discussed the importance of directing questions about the *counts by ones* strategy to the students who had not interpreted the problem correctly in the interviews. The teachers conjectured that students would benefit from identifying how the cards and friends are represented with pictures or manipulatives and explicitly connecting that information back to the problem.

Next, the teachers determined that the second strategy they wanted the class to focus on was a *counts by tens* strategy in which the student represented the 34 cards with base-ten blocks or drawings, showing 3 ten rods and 4 ones. This strategy was selected so that students could be asked to observe what was the same and different about this strategy and the *counts by ones* strate-

Strategy category	Ideas of How the Strategies Might Present During the Lesson
Counts by ones	<p>A. Students might draw 34 Pokémon cards, circle groups of ten, and attach a stick person to each group.</p> <p>B. Students might count out 34 snap cubes by ones and use them to make rods of ten.</p>
Counts by tens	<p>C. Students might build a representation of 34 with three base-ten rods counting, "10, 20, 30," and four unit cubes, "31, 32, 33, 34," then identify each ten rod as being one friend.</p> <p>D. Students might build a representation of 34 without explicitly counting by tens. They might know that the 30 in 34 can be built with 3 tens rods and simply pull out the rods. Then they might place each rod with some other picture or object to represent three friends.</p> <p>E. Students might use repeated addition ($10 + 10 \rightarrow 20 + 10 \rightarrow 30 + 4 \rightarrow 34$) and recognize that each ten represents the Pokémon cards for one friend.</p> <p>F. Students might use repeated subtraction ($34 - 10 = 24$, $24 - 10 = 14$, $14 - 10 = 4$) and explain that the subtracting of each ten is Royal giving 10 cards to a friend.</p>
Direct place value	<p>G. Students might immediately answer "three," with the explanation that three tens is 30.</p>
Did not solve with a valid strategy	<p>H. Students might add $34 + 10$ or subtract $34 - 10$.</p> <p>I. Students might represent 34 with base-ten blocks and then count the ten rods, "10, 20, 30," and answer 30.</p>

Figure 2. Strategies teachers anticipated that students might use for the Royal's Pokémon cards problem.

gy. The teachers agreed that they wanted to draw out how tens can be used to think about this solution, emphasizing the relationship to the problem context (representation of cards and friends) and also that ten ones are the same as one ten.

Within the lesson's 30-minute time frame, the teachers did not anticipate being able to discuss additional strategies, but they noted that, if time allowed, they might choose to have students examine a different *counts by tens* strategy, perhaps one involving repeated addition or repeated subtraction. The teachers reasoned that these strategies would help students move their thinking beyond strategies requiring manipulatives or pictures.

Strategy for differentiation to meet the needs of all students in the class

This lesson was developed to be a whole-class discussion in which students would examine and re-

flect on the mathematics used in their classmates' ways of solving the Royal's Pokémon cards problem. The teachers identified the student strategies to be included in the discussion to ensure inclusion of strategies at multiple levels of sophistication and sequenced the strategies in order of abstraction, placing the most concrete strategy first. This sequencing was intended specifically to support students who had not made sense of the problem in the interview. The teacher intentionally directed questions about the first strategy to students who had not previously understood the intended interpretation of the problem. As the discussion progressed to more abstract solutions, the teacher wanted to challenge students to make sense of the variety of ways to represent and think about the components of the problem.

Notes on what to notice about student thinking

During the lesson, the teachers wanted to pay close attention to how the students initially

solved the Royal's Pokémon cards problem. They wondered whether working with the number 34 (rather than the 67 in the interview) would make a difference in students' abilities to generate a valid strategy. They also wanted to pay close attention to the extent to which students were able to make sense of their classmates' strategies that were different from their own. Finally, the teachers wanted to look for evidence of students' engagement with the place value ideas that were central to the lesson. In particular, the teachers wanted to focus on the extent to which the students employed an understanding that ten ones are the same as one ten in their strategies and their explanations of the strategies of others.

Lesson Plan

This lesson took place in a second-grade classroom and involved the second-grade students in solving and discussing student-generated strategies for the following problem: *Royal has 34 Pokémon cards. He wants to give 10 cards to every friend. How many friends can he share with?*

In planning this 30-minute lesson, the teachers established the following learning goals:

Students will understand how the problem context of a measurement division problem involving groups of ten is related to different student strategies.

Students will understand the relationship between tens and ones in a two-digit number, specifically that ten ones are simultaneously one ten.

As the lesson opened, the teacher stimulated student interest by asking the second graders to raise their hands if they liked Pokémon cards. Many hands shot in the air, and the teacher revealed that she had brought a story problem for the class to solve that involved someone in the class giving Pokémon cards to friends. With students leaning in, the teacher verbally presented the word problem in an animated, storytelling style:

Our friend Royal has 34 Pokémon cards at home. He has so many friends, and he is so nice. Royal wants to give 10 cards to each of his friends. But he doesn't have enough cards to share with everybody. How many friends is he going to be able to share ten cards with?

Next, the teacher elicited help from the class with writing the important facts from the story on a sheet of chart paper. She asked students to offer answers to questions such as:

- What does Royal have in the story?
- Do you remember what Royal wants to do with those cards?
- Do you remember how many cards he wants to give to every friend?
- What is it that we need to figure out?

As individual students responded to each question, the teacher sought agreement from the rest of the class and created a written record of the problem (see Figure 3).

Next, the students were directed to work with a partner to retell the story about Royal and to make sure they understood the important information in the problem including what this problem wanted them to figure out. Following this exercise, students got to work on individually

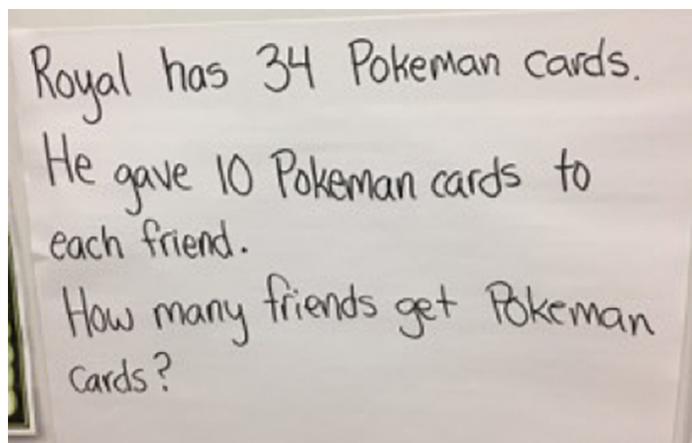
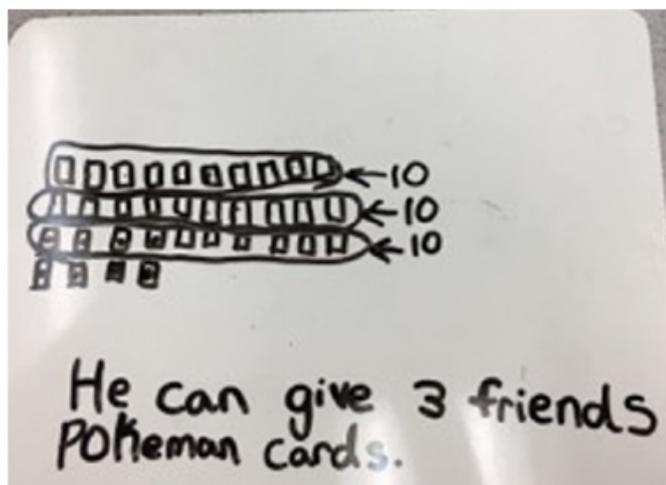


Figure 3. The public record of Royal's Pokémon cards problem that was created with input from second-grade students.

Osanna's Solution



Royal's Solution

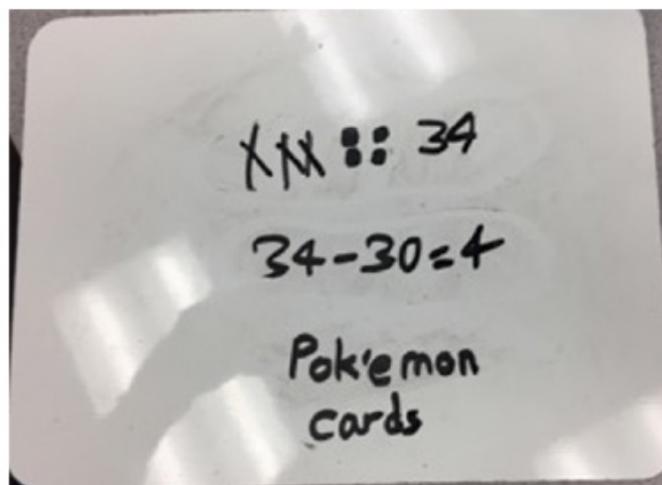


Figure 4. The two student work samples selected for focus in the class discussion.

solving the problem. Around the room, students approached the problem using a variety of tools including snap cubes, base-ten rods, and white boards (for drawing). A few students determined their answers without use of tools. Regardless of approach, every student used pictures and/or words to create a record of his or her strategy on a small white board.

As students worked, the teacher circulated and made note of the strategies students were using (e.g., *counts by ones*, *counts by tens*, and *direct place value*). She also posed questions to a few students to improve their understanding or stimulate their thinking. Through this process, the teacher selected the strategies of two students—Osanna and Royal—that she wanted to focus on with the whole group in the class-discussion phase of the lesson. The written records of these two students' strategies are presented in Figure 4.

After approximately ten minutes of work time, all students had a solution to the Royal's Pokémon cards problem, and most had created a record on a whiteboard. The teacher directed students to join her on the carpet with their whiteboards but not the markers. She reviewed the details in the Royal's Pokémon cards problem, and asked students to hold up their white boards so she could

see their solutions. At this point, the teacher moved her gaze slowly around the room to survey the students' boards. She noted that she was seeing some different answers and told the class that they were going to work together to figure this out. Next, she asked Osanna to bring her whiteboard to the front.

Osanna's strategy

With Osanna standing at the front and her white board propped on the marker ledge, the teacher directed the class to study Osanna's board (see Figure 4) and make a conjecture about what Osanna did first. After a few moments of wait time, several hands were in the air. The teacher called on one student who indicated that he thought she drew 34 cards as her first step. Others agreed, and Osanna confirmed that drawing 34 cards was the first thing she did. At this point, the teacher had the class engage in choral counting (by ones) for the stated purpose of making sure that Osanna had drawn 34 cards.

Next, the teacher turned to Osanna and said, "Osanna, I see that you put a ring around some of the Pokémon cards. Why did you do that?" Osanna timidly indicated that she had circled tens. At this point, the teacher asked multiple children to restate what Osanna had said (that she cir-

Next, the students were directed to work with a partner to retell the story about Royal and to make sure they understood the important information in the problem including what this problem wanted them to figure out.

pled tens), and she prompted the class to think about *why* Osanna put rings around groups of ten Pokémon cards. Then, the teacher called on Jeremiah, a student who had not interpreted the problem correctly in his personal solution. Jeremiah appeared to think for a few seconds and then indicated that he wasn't sure why Osanna had circled the cards. The teacher told Jeremiah and the class that it was a hard question, and that everyone would need to keep thinking about it. She reiterated the question to the whole class, "Why did Osanna put a ring around these ten cards?" and she sat back with a thinking expression. After a long wait time, the teacher called on a second student, Jonathon, who explained that the cards in a ring were for one friend.

The teacher dramatically asked the class, "What did Jonathon say? Why did she put a ring around these ten Pokémon cards [*gesturing to the top ring of ten*]?" She called on two different students who both indicated that those were the cards for one friend. Next, the teacher asked about the second and third row of ringed cards in Osanna's solution, "Why did she circle these cards?" Multiple students explained that each ring of ten cards would go to a different friend. At this point, the teacher turned back to Jeremiah, the child who had been previously unable to explain the rings of ten. She said, "Jeremiah, if you were one of the friends, which cards would you get?" Jeremiah responded that he would take the four cards at the bottom. To this, the teacher responded, "But, wait a minute...I thought Royal gives friends ten cards." After a moment of consideration, Jeremiah indicated that he would take the cards in the top row. Following acknowledgement of this response, the teacher thanked Osanna for sharing her strategy and directed her to take her seat on the carpet (but leave her white board on the ledge); and she requested that Royal bring his white board to the front.

Royal's strategy

When Royal came to the front of the class, he displayed his whiteboard (see Figure 4) and explained, "First, I made 34, and then I saw that I could give ten to three friends." The teacher di-

rected Royal to stay at the front while she asked the class some questions. First, she asked the class if Royal had drawn 34 Pokémon cards. Some students indicated “yes” and others “no.” The teacher prompted the students to keep looking and to try and figure out what Royal was thinking when he made his picture. Jeremiah, the same child who struggled to make sense of the problem earlier, raised his hand. When called on, Jeremiah tentatively said, “I think maybe the three lines are his tens and the four dots are his ones.” The teacher recreated Royal’s picture of three lines and four dots on the board and invited the class to help her count aloud using Jeremiah’s idea. The class choral counted the lines by tens, “10, 20, 30,” and the four dots by ones, “31, 32, 33, 34.”

At this point, the teacher said, “Osanna drew 34 Pokémon cards, and Royal drew 34 Pokémon cards. What’s different? Turn to a partner and talk about what’s different about how they drew it.” After a minute or two of partner time, the teacher called on students to share ideas. One student observed that Osanna had drawn ten single cards with a circle, and Royal just drew a line for ten. Jeremiah echoed the same idea, noting the Osanna made ones and “squeezed them together” to make tens, whereas Royal just made tens. A third student observed that Osanna’s board contained a word sentence, and Royal’s board had a number sentence.

Piggybacking on this comment about the number sentence on Royal’s board ($34 - 30 = 4$), the teacher said to Royal, “You wrote 34 minus 30 equals 4. Where did you take away 30 in your picture?” In response, Royal gestured to each of the three lines in his picture. The teacher then asked Royal why he took the 30 away. Royal responded that it was because he gave 30 cards to friends. Referring back to the word problem, the teacher noted that the question was, “How many friends get Pokémon cards?” Royal identified that three friends get Pokémon cards, pointing at each line and counting, “One, two, three.”

After thanking Royal and directing him to take his seat, the teacher engaged the class in a line of

questioning to emphasize the different representations of tens in the pictures made by Osanna and Royal. First she asked, “How many Pokémon cards is Royal giving each friend?” The class confidently responded with the answer ten. Next, she asked, “If you are one of Royal’s friends, where are the ten Pokémon cards that you could get?” After several seconds of wait time, the teacher called on multiple students to respond. The first three students to respond identified a row of ten cards in Osanna’s picture. The teacher ringed these groups in different colors, labeling them with a 1, 2, and 3 to make clear that each would go to a different friend. Next the teacher invited the class to identify the Pokémon cards that would go to each friend in Royal’s picture. As students responded, the teacher used the same color-coding and labeling used on Osanna’s picture, emphasizing the relationship between the two representations (see Figure 5).

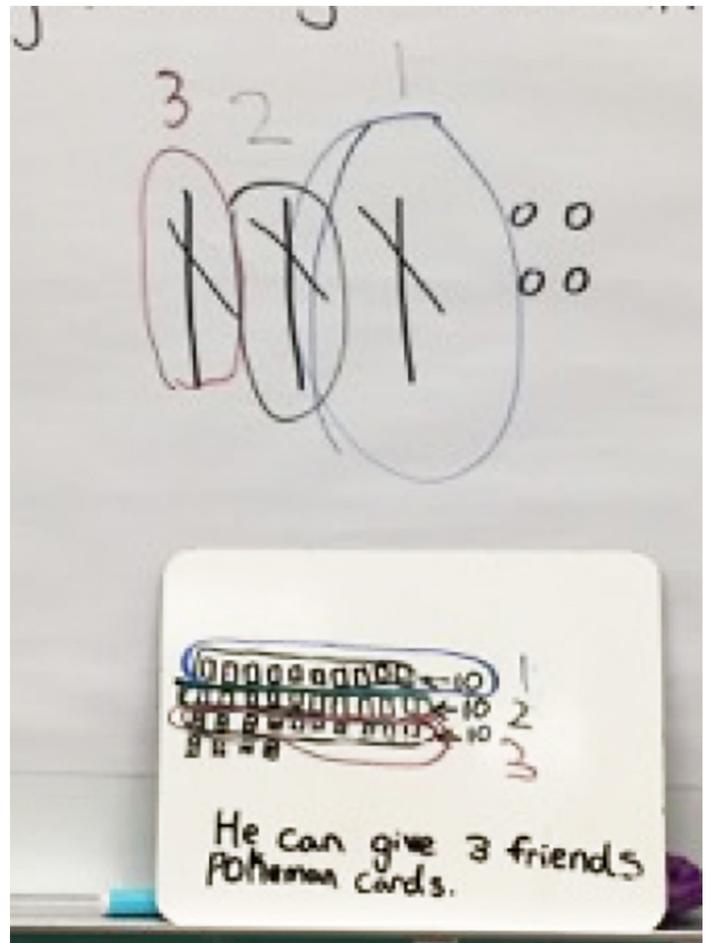


Figure 5. The teacher used color coding and labeling to emphasize the different representations of tens in students’ solutions.

To wrap-up the lesson, the teacher praised students for thinking hard about Osanna and Royal's solutions and the ideas shared by their classmates in the discussion. She also offered a summary of the class findings, stating that the class had noticed that ten could be represented in different ways and that the tens in both pictures represented ten cards and what one friend would get.

Reflection

What We Learned About the Students

With two exceptions, all of the students approached the Royal's Pokémon cards problem in a way that matched the problem context. The teachers attributed the significant decrease in inability to solve with a valid strategy (from that in the interviews) to the easier numbers and the instructional techniques used by the teacher to help students understand the problem. Specifically, the design of the word problem involved the name of a student in the class, and the teacher attempted to make the problem personal to the students. She presented the word problem in an engaging, storytelling style and then had the students work together to retell the story while she created a public record. Then, before students started working on a solution to the problem, they were asked to discuss the important information in the problem with a partner. The teachers conjectured that these moves prompted students to process the word-problem situation in a deeper way.

As in the interview, the most common strategy employed by students was *counts by tens*, in which students used pictures or manipulatives to represent the 34 with three tens and four ones. After representing 34, some students recognized each group of ten cards as being for one friend and indicated the correct answer (3 friends). Other students counted their tens, "10, 20, 30," and reported the answer (incorrectly) as 30. The teachers conjectured that these students had difficulty seeing each set of ten cards as also being for one friend. They discussed how this struggle highlights the difficulty students have in simultaneously thinking of ten ones as also being one

ten. The teachers agreed that the class discussion of Osanna and Royal's solutions did a good job of pressing students to think about this relationship.

Reflections on the Lesson

One aspect of the lesson that the teachers discussed at length after the lesson is the way the teacher used strategies to promote student engagement in examining the solutions of their classmates. Of particular note was the teacher's success with engaging Jeremiah, a child who had not produced a valid solution on the interview problems or the Royal's Pokémon cards problem. The teacher revealed that, during the class discussion, she intentionally asked questions of Jeremiah and another student whom she noticed had not made sense of the problem. Rather than calling on students whom she was pretty sure already knew the answers to certain questions, she wanted to direct her questions to the students who "needed to think about them." She noted that it was her plan to engage particularly the struggling students through the discussion of Osanna's solution, because it was the most concrete.

When Jeremiah could not answer a question (i.e., when he responded that he wasn't sure why Osanna had circled the ten cards), the teacher deliberately validated his struggle by acknowledging that she had asked a hard question, and she appropriated his struggle to the whole class, saying, "We are all going to have to keep thinking about this." Then, after a classmate had explained the answer to the question originally posed to Jeremiah, the teacher had multiple students reexplain why Osanna had put a ring around each set of ten cards. At this point, she came back to Jeremiah and tried to engage him with a more personal approach, "Jeremiah, if you were one of the friends, which cards would you get?" Jeremiah initially struggled with his answer but ultimately did identify a group of ten cards that could be for him. Then, in the discussion of Royal's solution, Jeremiah volunteered a valid explanation of Royal's picture, when he said, "I think maybe the three lines are his tens and the four dots are his ones." The teachers attributed Jeremiah's ultimate success to the teacher's persistent efforts to engage

him actively during the discussion, her use of wait time to allow space for “figuring things out,” and the way she continually encouraged students to listen to each other and keep thinking.

Looking forward, the teachers agreed that this class of second-grade students would benefit from more experiences with multiplication and measurement division problems involving groups of ten. After this experience with Royal’s Pokémon cards problem that involved finding the number of tens in 34, the teachers conjectured that the students might be ready to engage with a problem focused on finding tens in a quantity between 50 and 100. Through such an experience, the teacher might orchestrate class discussion around comparison of a *counts by tens* strategy with a *direct place value* strategy so as to stimulate greater focus and discussion on the meaning of the digits in a two-digit number.

The teacher revealed that, during the class discussion, she intentionally asked questions of Jeremiah and another student whom she noticed had not made sense of the problem.

What's Next?

Stories of teachers engaging in collaborative inquiry focused on using student thinking to inform instructional decisions

What's Next? is a collection of stories documenting professional development experiences shared by elementary teachers working collaboratively to study the complex process of teaching and learning mathematics. Each story in the collection describes practicing teachers studying the thinking processes of real students and using what they learn about those students to make decisions and try to help advance those students' understanding on that day.

The teachers in each story start by learning about how individual students are solving a set of mathematics problems. They use this freshly gathered knowledge of student thinking to develop near-term learning goals for students and a lesson plan tailored to specific students on that specific day. One of the teachers implements the planned lesson while the other teachers observe in real time. The teachers then gather to discuss and reflect on their observations and insights.

In these lessons, the practice of teaching is slowed way down. The stories tell of teachers who are studying student thinking and using that information to plan and implement instructional decisions at a pace that is much slower than it occurs in daily practice. The stories in this collection also depict many aspects in common with formative assessment and lesson study, both of which are a process and not an outcome.

The stories depict real situations that occurred in real time and include both successes and shortcomings. We hope that the stories may be studied and discussed by interested educators so that the lessons and ideas experiences of these teachers and instructional coaches may contribute to additional learning and sharing among other interested teachers.

Learn more about these and other stories at <http://www.teachingisproblemsolving.org/>

