Using Invented Algorithm Strategies to Solve Multidigit Word Problems

This story is a part of the series:

What's Next? **Stories of Teachers Engaging in Collaborative Inquiry Focused on Using Student Thinking to Inform Instructional Decisions**

What's Next?

Stories of teachers engaging in collaborative inquiry focused on using student thinking to inform instructional decisions

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Introduction

A group of teachers seeking to understand second-grade children's engagement with base-ten concepts interviewed a class, asked each student to solve a subtraction word problem, analyzed the student work and thinking, and planned a lesson. The lesson was designed to encourage the students to use self-generated mental strategies as they solve the word problem.

Relevant Florida Mathematics Standards

MAFS.2.NBT.2.5 Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.

Background Information

This investigation was informed by chapters two, three, six, and seven in Children's Mathematics: Cognitively Guided Instruction (Carpenter et al., 2015). These chapters provide background on addition and subtraction problem types as well as strategies that students use to solve them. In addition, chapter six provides some background information on base-ten number concepts. Chapter seven provides additional information on children's strategies for dealing with multidigit numbers.

Carpenter, T. P., Fennema, E., Franke, M. L., Levi, L., & Empson, S. B. (2015). *Children's Mathematics: Cognitively Guided Instruction (2nd. Ed.).* Portsmouth, NH: Heinemann.

Analyzing Student Thinking

A group of teachers conducted one-on-one interviews with a class of second-grade students with the goal of obtaining a good understanding of each child's mathematical concepts related to subtraction and place value on that day. The students had access to base-ten blocks, pencils, and paper and were encouraged to use whatever tool or solution strategy they wished. The interviewing teacher read the problem to the student, and the reading of the problem could be repeated as needed. After the student provided an answer, the student was asked to explain how he or she solved the problem. They posed the following problem to each student.

Rob had 82 rocks. He lost 47 of them. How many rocks does Rob have now?

The following provides a summary of the working definitions of the student strategies that were used during the interviews.[1]

Multidigit Computation Strategies

A student who uses a *direct modeling* strategy represents each multidigit number in the problem using manipulatives or pictures to model the story in the problem and then counts the objects or pictures to determine the answer. In the Rob's rocks problem, a student using a *direct modeling* strategy might create a set of 82 objects and then take 47 of the objects away and count the remaining objects by ones. Figure 1 shows an example of a *direct modeling* strategy on the Rob's rocks problem.

A student who uses a *standard algorithm* strategy follows the traditional subtraction steps taught in the United States. For the Rob's rocks problem, a student might write the subtraction problem vertically with 82 on top and 47 underneath and then follow the traditional steps of subtracting the digits in the ones place—with borrowing from the tens place if necessary—followed by subtracting the digits in the tens place. Figure 2 shows the work of a student who successfully used the *standard algorithm*, and Figure 3 shows the work of a student who made errors in the *standard algorithm* process.

A student who uses an *invented algorithm* strategy demonstrates flexibility in thinking about

1 The descriptions of strategies presented here are the current descriptions used by our team, and we consider them to be fluid, as our understanding of these ideas continues to evolve. For a more detailed discussion of these terms, consider reading Carpenter et al. (2015).

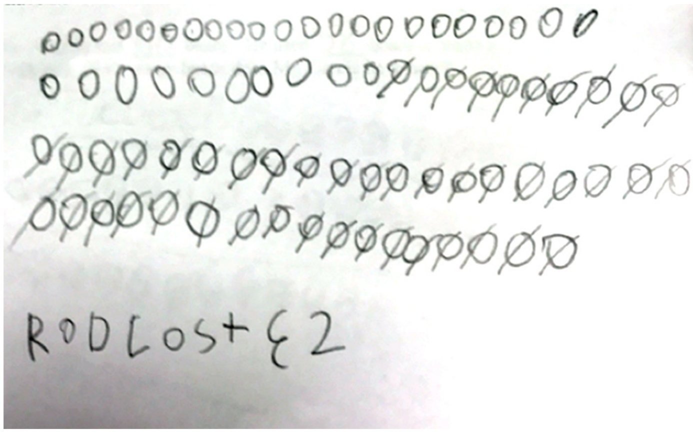

Figure 1. A direct modeling strategy for the Rob's rocks problem

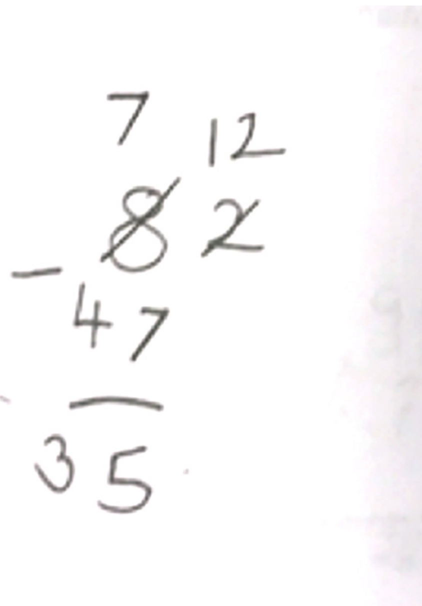

Figure 2. A student who used the standard algorithm strategy and got a correct answer

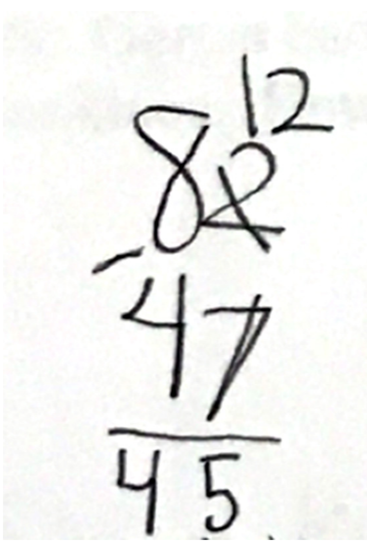

Figure 3. A student who used the standard algorithm strategy and got an incorrect answer

numbers by breaking them apart and putting them together in different ways without the use of manipulatives or pictorial models. For example, in the Rob's rocks problem, a student might subtract 40 from 82 and get 42 and then count back an additional seven to get a final difference of 35. Another *invented algorithm* strategy would be for a student to add three to both the 82 and the 47, keeping the distance between the two values constant, and then to subtract 50 from 85 mentally to get 35.[2]

Summary of Strategies Used by Students in This Class

After interviewing the students in the class and reflecting on the strategies the students used to solve the problems, the teachers sorted students' strategies for the Rob's rocks problem as shown in Figure 4.

The teachers began a discussion about how the students solved the problem by noting that most students used a *direct modeling* strategy. They also noticed that students showed some variation in how they used a *direct modeling* strategy. Some students used *direct modeling with tens*, whereas others used *direct modeling with ones*. A student who uses a *direct modeling with ones* strategy represents each multidigit number in the problem as a set of ones using manipulatives or pictures to model the story in the problem and then counts the objects or pictures to determine the answer. A student who uses *direct modeling with tens* represents each multidigit number in the problem using manipulatives or pictures that reflect the base-ten structure of the number system (e.g., with base-ten blocks or base-ten pictures). Then the student counts the objects or pictures by tens and ones to determine the answer.

After discussing these variations in *direct modeling* strategies, the teachers discussed their observation that no students in this classroom had used *invented algorithms*. After thinking about what is

2 Students may use many other strategies that involve *invented algorithms* while solving multidigit word problems. For more information about *invented algorithms*, read chapter seven of Carpenter et al. (2015).

Direct Modeling	*Standard Algorithm*	*Invented Algorithm*
Gerald	Jaylen	
Trenton	Eliana	
Tristan	Jazmin	
Jariely	Angelina	
Lorenzo		
Luna		
Dresden		
Jazmin		
Angelina		
Eliana		
Stephanie		
Angel		
Karla		

Note. Jazmin and Eliana are listed in both the *direct modeling* and *standard algorithm* categories. The interviewers of both students observed them to use both a *direct modeling* and a *standard algorithm* strategy to solve the problem.

Figure 4. Teachers' classification of students' strategies for the Rob's rocks problem

entailed in an *invented algorithm*, the teachers noticed that a typical subtraction word problem, such as Rob's rocks problem, in which the result is unknown may not encourage students to use *invented algorithms*. If they follow events in the story, the students are likely to start with 82, in either numerals or objects, and take away 47. The teachers discussed other word problem types and decided that a *join, change unknown*[3] word problem would encourage students to think about subtraction as the difference between the two values and might encourage the use of *invented algorithms*. They also concluded that using a *join, change unknown* word problem would make using *direct modeling* strategies more difficult. Change-unknown problems can be solved thought either addition or subtraction, and this ambiguity might also increase the number of strategies used to solve them.

After the discussion of strategies, the teachers decided on the following learning goal:

Encourage students to investigate and use invented algorithm *strategies.*

Planning for the Lesson

The teachers felt strongly that a join, change unknown word problem would encourage the type of thinking they wanted to see. After deciding on the problem type, the team turned their attention to the number choice. They initially thought that 60 and 30 were good choices, because it would be easy for students to see the distance between the two numbers. However, they were worried that many of the students that were still using a *direct modeling with ones* or *direct modeling with tens* strategy would rely on those strategies with those numbers. They then decided to investigate two numbers that were closer together. They ultimately decided on 28 and 40. They thought that this pair of numbers would encourage students to think about how much more they needed to add to 28 to get to 40 and would discourage students from using a *direct modeling* strategy. The teachers considered these numbers likely to encourage students to add two and then add ten or add ten and then add two to get to 40.

3 A *join, change unknown* word problem is a joining or combining situation where the starting amount and the result are known but the amount of change is unknown.

The teachers wrote the following problem for the lesson:

> *Jazmin has 28 beads. How many more beads does she need to have 40 beads altogether?*

Lesson Plan

On the basis of how the students solved the Rob's rocks problem, the teachers determined they would use the following learning goal to guide decisions during the classroom lesson:

> *Encourage students to investigate and use* invented algorithm *strategies.*

The lesson plan follows.

1. Present the new problem to the class and consider using some of the strategies that follow to support students' comprehension.

   a. Read the problem aloud: *Jazmin has 28 beads. How many more beads does she need to have 40 beads altogether?*

   b. Possible strategies to support comprehension:

      i. Reread the problem as needed.

      ii. Have students discuss the important details of the problem with a shoulder partner.

      iii. Ask the class questions to probe understanding of the problem. For example, "How does the story start? What is Jazmin looking for?"

   c. Direct students to think of a strategy for solving the problem and to give you a signal (e.g., thumbs up on chest) to indicate when they have ideas.

      i. Once most students indicate they have strategies in mind, send them back to their desks to start solving the problem.

The teachers were impressed with these changes to the students' strategy choices and concluded that the join, change unknown word problem and the teacher's guiding questions influenced the student's strategy choices.

    ii. Remind students they can use whatever tools they want and whatever method they feel most comfortable using. Encourage the students to record their solution strategies so that they can share them with the class.

2. Circulate and observe what students are doing. Allow enough time for students to solve the problem in whatever way they choose and to record their thinking in numbers, pictures, or words. Attend to the following:

    a. Do students use a strategy similar to or different from the one they used on the interview problem?

    b. For those who model the problem with objects or pictures, do they choose to model and count by tens when possible?

    c. Do students notate their strategy symbolically (e.g., with an equation)? If so, consider noting how.

3. Ask probing questions to promote student understanding of the problem and to improve your understanding of their strategies:

    a. For students who struggle to interpret the problem or get started with a strategy, ask, "What do we know about Jazmin and her beads? How many does she have? How many does she want to have?"

    b. For students working on a strategy, ask, "How are you thinking about the problem?"

4. For the whole-class discussion on ways to solve the problem, identify 2–4 student strategies to be shared in class discussion that will showcase students who used *invented algorithm* strategies. Consider having a student who used a *direct modeling with tens* strategy to share first, to help ensure that all students are able to understand how the student solved the problem. After that strategy, invite students who were thinking about this problem using *invented algorithm* strategies to share their strategies. If no students used an *invented algorithm* strategy in the class, consider showing one to the students that you create and ask them to investigate how to interpret it.

5. Gather the whole class and establish the purpose of the discussion: "During this whole-group discussion, I'd like some of our classmates to share their thinking on this problem about Jazmin and her beads. While your classmates are presenting their ideas, pay close attention to how they solved the problem and consider what questions you have about their work."

6. Invite selected students to share their strategies one at a time. After each student shares, ask the class, "What questions to do you have for [insert student name]?" Provide probing and follow up questions as necessary to highlight the salient portions of the student thinking. You may also consider choosing two *invented algorithm* strategies and asking the class to discuss how they are alike and how they differ.

7. End the lesson by asking students to share with a partner one idea they learned today. Listen closely to what they say. Invite a few students to share what they learned with the class.

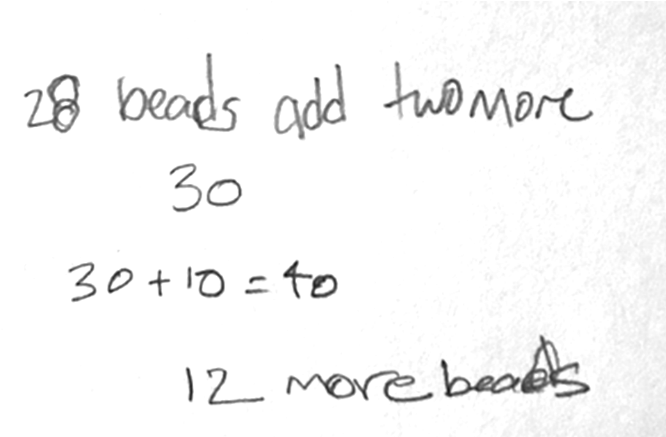

Figure 5. Trenton's invented algorithm *strategy*

Reflection

After this lesson was implemented, the teachers noticed that a significant number of students used *invented algorithms* on the lesson's problem. Many students were able to think about the distance between 28 and 40 as 28 + 2 = 30, and 30 + 10 = 40, so the answer would be 10 + 2 = 12. This strategy can be seen in Trenton's work (Figure 5).

Fewer students added ten first. Only one student, Eliana, used this strategy. Her work can be seen in Figure 6.

The teachers were impressed with these changes to the students' strategy choices and concluded that the *join, change unknown* word problem and the teacher's guiding questions influenced the student's strategy choices. They also discussed the importance of the sharing of student strategies in the lessons. The teachers thought it was very important for students like Eliana and Trenton to share their thinking in the lesson, because it helped other students make sense of these *invented algorithms* and encouraged them to try using such strategies in the future. It also provided opportunities for teachers to help students express their ideas in writing by means of mathematical notation.

The teachers also discussed the next steps for this group of students. They initially discussed whether some students might be ready to investigate *join, change unknown* word problems with numbers greater than 100. The conversation then turned to how to use *join, change unknown* word problems to highlight the relationship between addition and subtraction. While this topic wasn't a large focus in this lesson, the teachers concluded that it could be highlighted in future lessons by comparison of student strategies that use additive strategies to those of other students who might use subtractive strategies to solve the problems.

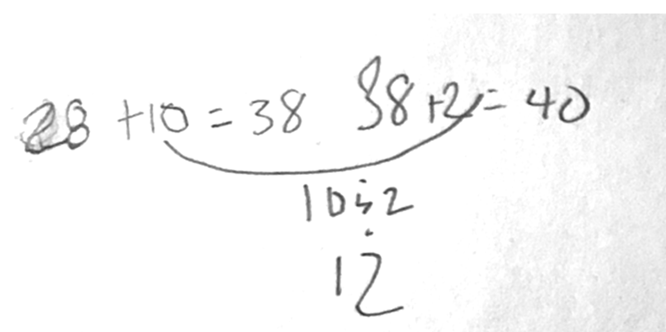

Figure 6. Eliana's invented algorithm *strategy*

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Stories of teachers engaging in collaborative inquiry focused on using student thinking to inform instructional decisions

What's Next? is a collection of stories documenting professional development experiences shared by elementary teachers working collaboratively to study the complex process of teaching and learning mathematics. Each story in the collection describes practicing teachers studying the thinking processes of real students and using what they learn about those students to make decisions and try to help advance those students' understanding on that day.

The teachers in each story start by learning about how individual students are solving a set of mathematics problems. They use this freshly gathered knowledge of student thinking to develop near-term learning goals for students and a lesson plan tailored to specific students on that specific day. One of the teachers implements the planned lesson while the other teachers observe in real time. The teachers then gather to discuss and reflect on their observations and insights.

In these lessons, the practice of teaching is slowed way down. The stories tell of teachers who are studying student thinking and using that information to plan and implement instructional decisions at a pace that is much slower than it occurs in daily practice. The stories in this collection also depict many aspects in common with formative assessment and lesson study, both of which are a process and not an outcome.

The stories depict real situations that occurred in real time and include both successes and shortcomings. We hope that the stories may be studied and discussed by interested educators so that the lessons and ideas experiences of these teachers and instructional coaches may contribute to additional learning and sharing among other interested teachers.

Learn more about these and other stories at **http://www.teachingisproblemsolving.org/**