Comparing Denise and Raquon's Strategies to Solve a Join (Change Unknown) Word Problem

This story is a part of the series:

***What's Next?* Stories of Teachers Engaging in Collaborative Inquiry Focused on Using Student Thinking to Inform Instructional Decisions**

What's Next?

Stories of teachers engaging in collaborative inquiry focused on using student thinking to inform instructional decisions

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Introduction

This narrative tells the story of a group of teachers studying how to teach children to use place-value concepts to find the difference between two numbers. This lesson focuses on two students sharing their strategies while solving a word problem with multidigit numbers. It emphasizes finding connections between pictorial representations and counting strategies for a deeper understanding of the relations between numbers.

Relevant Florida Mathematics Standards

MAFS.2.NBT.2.5 Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.

Background Information

This investigation was strongly informed by chapters two, three, six, and seven in *Children's Mathematics: Cognitively Guided Instruction*. These chapters provide background information about addition and subtraction problem types as well as strategies that students use to solve these types of problems. In addition, chapter six discusses base-ten number concepts. Chapter seven provides additional information on typical strategies children use when solving problems involving multidigit numbers.

Carpenter, T. P., Fennema, E., Franke, M. L., Levi, L., & Empson, S. B. (2015) *Children's Mathematics: Cognitively Guided Instruction (2nd ed.)*. Portsmouth, NH: Heinemann.

Analyzing Student Thinking

In one-on-one interviews, a group of teachers posed a series of problems to second-grade students in a single class. The students were interviewed individually with the goal of obtaining a thorough assessment of each child's mathematical thinking related to the difference between numbers and place-value concepts on that day. The students had access to base-ten blocks, pencils, and paper during the interview and were encouraged to use whatever tool and solution strategy they wished. The interviewing teacher read each problem to the student and repeated reading the whole problem as needed. After providing an answer, the student was asked to explain how he or she solved the problem. The following four problems were posed, one at a time.

> *Problem A. Solve 301 – 299 (presented vertically).*
>
> *Problem B. Rob had 82 rocks. He lost 47 of them. How many rocks does Rob have now?*
>
> *Problem C. Amanda has ____ shells. Zoe has ____ shells. How many more shells does Zoe have than Amanda?*
>
> *Problem D. Mr. Garcia had ____ beads. He used ____ of those beads to make a necklace that he gave to his mother. How many beads does Mr. Garcia have now?*

Teachers selected numbers for problems C and D after determining whether students were successful with problems A and B. If the students were not able to solve problem B correctly, then the teachers chose two-digit numbers less than 20. If, instead, the students had no trouble solving the first two problems, the teachers chose numbers similar to those of problem B. In all cases, they chose numbers in which the ones digit in the minuend was less than the ones digit in the subtrahend, so as to determine whether students used the standard algorithm to solve. Some examples of numbers selected by teachers during the interview include (64, 29) and (50, 17). If a student used the standard algorithm for problems B, C, and D, the teacher asked the student whether the problem could be solved in another way.

Multidigit Computation Strategies[1]

A student who uses a *direct modeling with ones* strategy represents each multidigit number in the problem as a set of ones using manipulatives or pictures and then counts the objects or pictures to determine the answer. In Problem B, for example, a student might create a set of 82 objects and then remove 47 of the objects from the set and count the remaining objects by ones.

Most of the students who used a nonvalid strategy added the two numbers given in the problem.

A student who uses a *counting* strategy will do so without physically representing each quantity in the problem. Fingers, objects, or tally marks are often used to keep track of the number of counts. In Problem B, a student using a *counting by ones* strategy might say "47" and proceed to count forward by ones until he or she arrives at 82. The student might use his or her fingers to keep track of the 35 numbers counted from 47 to 82.

A student who uses a *directing modeling with tens* strategy represents each multidigit number in the problem using manipulatives or pictures that reflect the base-ten structure of our number system (e.g., with base-ten blocks or base-ten pictures). The student then counts the objects or pictures by tens (and ones) to determine the answer. In Problem B, a student using a *direct modeling with tens* strategy might use base-ten blocks to model 82 by representing it with eight ten rods and two unit cubes. The student might then break a ten rod into ten unit cubes to yield seven ten rods and 12 unit cubes. He or she might then take away 47 as four ten rods and seven unit cubes and count the number of remaining objects, either by ones or by tens and ones. For example, the student might say, "10, 20, 30, 31, 32, 33, 34, 35." (The student might also count the ten rods by ones and consider three ten rods to be 30.) If the student counts from 1 to 35 by ones, the student's strategy is still considered *direct modeling with tens*, because the student created a representation involving groups of tens and ones.

A student who uses a *combining the same units* strategy operates on the tens and ones separately without the use of manipulatives or pictorial representations, and the partial sums or differences are then combined to yield a final result. In Problem B, the student might think about subtracting four tens from eight tens to make 40 and subtracting seven from two to make negative five. The student might then say, "40 minus 5 is 35." If the student says that he or she cannot take seven from two and must decompose the 40 into 30 plus 10, the student is still considered to be *combining the same units*, because the student is breaking the numbers into values based on place value.

1 The descriptions of strategies presented here are the current descriptions used by our team, and we consider them fluid, as our understanding of these ideas continues to evolve. For a more detailed discussion of these terms, consider reading Carpenter et al. (2015).

Direct modeling (DM) with ones	Direct modeling with tens	Count forward/ count back	Incrementing	Combination of strategies	Nonvalid
	Joshua Magaly Harmony Alexia Morgan	Scarlett	Magaly Raquon	Alyssa (DM with ones and Count forward) Xavier (DM with tens and Count forward)	Denise Jeremy Jordyn Carly Andre

Figure 1. Teachers' classification of students' strategies for problems A and B.

The student who uses an *incrementing* strategy determines the answer by increasing or decreasing partial sums or differences without the use of manipulatives or pictorial representations. In Problem B, a student using an *incrementing* strategy might say, "47 plus three is 50, 50 plus 30 is 80, 80 plus two is 82, so what I added was three plus 30 plus two, which is 35."

Summary of Student Strategies

After interviewing the students in the class and reflecting on the strategies the students used to solve the problems, the teachers sorted students' valid strategies for Problems A and B as shown in Figure 1. The students who used nonvalid strategies were listed in the last column. Some students are listed twice, because the sorting was completed for two problems, and some students used a different strategy for each problem.

Most of the students who used a nonvalid strategy added the two numbers given in the problem. Andre and Jordyn attempted to combine same units to solve Problem B, but they added the two given numbers instead of subtracting. Xavier, Jeremy, Denise, and Harmony all attempted to use a standard algorithm to solve Problem A, but they added the numbers instead of subtracting. Denise attempted to solve the problem using a pictorial representation, but she represented the subtrahend and minuend separately, which is a common mistake students make when attempting to use a pictorial representation to solve a problem involving subtraction.

Before giving the problem to students, the teachers anticipated that some students would use these strategies. No students used valid *direct modeling with ones, counting, combining same units strategies*, or the *standard algorithm*, so those categories were removed from Figure 1.

Learning Goals

On the basis of how the students solved Problems A and B, the teachers wrote the following overarching goal for the new lesson:

> *Students will make sense of the context of a word problem, use a strategy they understand to solve the problem, and think about the reasonableness of their answers.*

Planning for the Lesson

The teachers developed the following word problem to use as the basis of the lesson.

> *Harmony had 42 rocks. She found some more rocks on the way home. Now she has 83 rocks. How many rocks did she find on her way home?*

Rationale for the problem selected

The teachers chose to use a *join (change unknown)*[2] problem. They decided to pose this

2 More information about this type of problem can be found in *Children's Mathematics: Cognitively Guided Instruction* (Carpenter et al., 2015).

type of problem to allow for the students' using either addition or subtraction to solve the problem. The numbers 42 and 83 were chosen to provide an opportunity for students to use mental strategies such as *combining like units* and *incrementing*. Because the students had just completed a science unit on rocks, the teachers determined this would be a good context for the new problem.

While students were solving the new problem, the teacher would identify students to ask to share their thinking with the rest of the class during the discussion segment. The teacher would look specifically for students who had moved from *direct modeling* to *counting* strategies or to more abstract, mental strategies such as *incrementing* or *combining like units*. The teacher would also plan to make note of any other students who used a strategy different from what their previously one.

Overall, seven students used a strategy to solve the lollipop problem that differed from the one they used in similar problems previously.

As shown in Figure 1, several students did not understand the context of the problem during the interviews. The teacher, therefore, planned to pay close attention during the new lesson to those students and to ask students to summarize what they understood about the situation in the problem.

Lesson Plan

On the basis of their observations of how the students solved Problems A and B, teachers created the following goal to focus their instructional decisions during the classroom lesson:

Students will make sense of the context of a word problem, use a strategy they understand to solve the problem, and think about the reasonableness of their answers.

1. Invite students to sit on the floor. Remind them of the behavior expected of them while they sit on the floor.

2. Say, "You solved some problems earlier, and you used several different strategies to solve them. We are going to solve another problem. You can use the base ten blocks, paper, or your brain to solve it. I am going to ask some of you to share your thinking. For today's problem, the story has to do with one of you. Today's problem is about Harmony. So when we read the problem, I want you to picture Harmony in your head. Our problem has something to do with science."

3. Ask the students, "Who is today's problem about?" Wait for students to think about an answer and invite a student to answer.

4. Say, "Don't pick up your pencil yet. I want you to just think about the problem and make a picture in your head."

5. Display the problem for students to see and read the problem aloud to the class.

Harmony had 42 rocks. She found some more rocks on the way home. Now she has

83 rocks. How many rocks did she find on her way home?

6. Say, "Harmony had 42 rocks. Form a picture of that in your mind. She found some more rocks. Picture that in your mind."

7. Invite a student to read the problem aloud to the class.

8. Ask the class, "Who is this problem about? What does Harmony have? How many rocks did Harmony have? What happens next in the problem? What do we need to figure out in the problem?"

9. Provide time for students to think, and invite one student to answer. A student who understands the situation in the problem might say something like, "She finds some rocks until she gets to 83."

10. Ask the class, "When she gets home, what does she have?"

11. Hold a discussion about how Harmony knows she has 83 rocks when she gets home.

12. Ask the class, "What do we need to figure out in this problem?"

13. Invite several students to answer. They will probably answer something like, "How many rocks she found on her way home."

14. Say, "You are going to go back to your seats and solve this problem. You can use base-ten blocks, paper, and your brain to solve the problem. Quietly go back to your seats and get started solving. I'll be walking around asking you some questions."

15. Circulate around the room and observe the strategies the individual students are using to solve the problem. Interact one-on-one with some of them to ask any clarifying questions. If students are using nonvalid strategies, discuss the situation in the problem with them.

16. After the students have had sufficient time to finish solving the problem, invite them back to the rug for a class discussion.

17. Say, "We saw a lot of different strategies today. Let's read the problem again."

Note. For the rest of the account, the narrative describes what happened in the actual lesson. Denise was identified by the teacher to share her strategy, because she had made great strides in making sense of the problem, and she used a strategy using base-ten manipulatives. Also, her strategy demonstrates a connection between addition and subtraction, which is a strength of using *join (change unknown)* problems in instruction.

18. The teacher read the problem aloud to the class and said, "Think about how you solved the problem. I'm going to ask Denise to come up and share how she solved the problem. While she shares, I want you to listen closely and think about how your strategy was similar to or different from Denise's strategy."

19. Denise was invited to the document camera to display her work. She said, "First, there were 42 rocks." Her model of the 42 rocks was displayed under the document camera to the class as shown in Figure 2.

20. The teacher interrupted to ask the class, "Why did Denise put 42 there?" Students were given an opportunity to answer and relate the number 42 back to the problem.

21. The teacher asked Denise, "What did you do next?" Denise placed eight ten rods and three unit cubes beneath her model of the 42 rocks as shown in Figure 3. She said, "So then I put 83, because she had 83."

22. The teacher then asked the class, "What do you think Denise is going to do next?" Before inviting them to respond, she asked, "Denise, what did you do next?"

Figure 2. Denise's initial model for the Harmony's-rocks problem.

Figure 3. Denise's model of both 42 and 83.

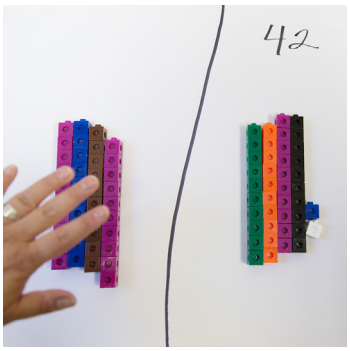

Figure 4. The teacher shows Denise's explanation on the board.

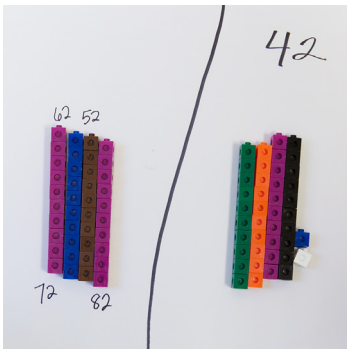

Figure 5. The teacher writes numbers corresponding to the students' verbal counting

23. Denise answered, "I added tens. Add one more, that's 50. Add another and that is 60. Add one more that is 70, and add one more and that is 80." Denise added four ten rods to the initial model of 42 rocks. The model of the 83 rocks was not referenced again after she initially modeled with eight ten rods and three unit cubes.

24. Denise then said, "So I started with 42 and now I need to take away the ones I added (referencing the four ten rods she added to the model of 42), so I started with 42 and then counted how many I took away, 10, 20, 30, 40." Denise pulled four ten rods away from the new model of 82, leaving 42 left.

25. The teacher asked Denise, "What was your answer when you solved?"

26. Denise answered, "40."

27. In Figure 4, the teacher drew a line separating the 42 that Denise said she started with and the 40 that Denise said she took away.

28. The teacher said, "She had 42 and added one more ten. What number would that be?"

29. Students responded, "52." The teacher said, "Keep going." As the students counted forward from 42 by tens, the teacher recorded the numbers in writing as shown in Figure 5.

30. The teacher said, "Denise, we ended at 82. Are we done? You have a friend who wants to tell you something."

31. A student told Denise, "You need to add one block."

32. The teacher said, "I want you to turn to your elbow partner. What did we understand by Denise sharing that strategy? Does it match what you did? Or is it different?"

33. After students had time to talk with their partners, the teacher directed the students' attention back to the large group and said, "I heard a lot of good strategies. Some of you said that one is still missing from Denise's model. I'm going to ask Raquon. Raquon, what numbers were you thinking in your head?"

34. Raquon said, "I started with 42 and then counted out four more ten blocks."

35. The teacher asked, "What did that give you?"

36. Raquon said, "82. But then I knew I needed to add one more block, because the problem said 83."

37. The teacher said, "So how many rocks did she get on the way home?"

38. Raquon said, "41."

39. The teacher asked, "Raquon, can you come up and show us where you see the 41?"

40. Raquon went to the screen and placed one unit cube with four the ten rods and said, "The 40 and the 1. She had 42 rocks, and that is what this side shows (pointing to the left side of the screen which shows four ten rods and two unit cubes). If I count these tens and start at 42, that is 52, 62, 72, 82. But the problem said she had 83 after walking home. So I needed to put one more unit cube here on this side (referencing the side that has only four ten rods). So now, if I count these cubes, I have 10, 20, 30, 40, and one more is 41."

41. The teacher directed the class to turn to their partners and tell them something new that was learned today about this problem.

Reflection

Many of the students demonstrated progress in their ability to share their own strategies and to relate their own strategies to those used by their peers. The classroom teacher noted that a couple of students who typically struggled to explain their strategies showed improvement on the day of the lesson.

The teachers thought that Denise still was not quite convinced that Raquon's answer was correct and her answer was not. All agreed that Denise needs to have more opportunities to solve similar problems to support her ability to make sense of her strategy in a way that will get her to the correct answer. Denise did make progress from the strategy she used during the initial interview. During the interview, Denise created two distinct sets corresponding to the two numbers in the problem when she was subtracting. Although she initially modeled both 83 and 42 in the lesson, she recognized that she had to add some tens to 42 and answered with how many tens she added. Although she did not see that she needed to add an additional unit cube, she did recognize and attempt to add tens to 42 in an effort to get to 83. The teachers agreed that she was beginning to make more sense of the problem.

The teachers agreed that this class was ready to solve similar problems involving numbers that could create a need for students to break apart groups of tens. They thought a *join (change unknown)* problem with the numbers 48 and 76 might be a good fit for these students. In addition, the teachers conjectured that students might benefit from more opportunities to share their strategies with the class in the future. Students might benefit from more opportunities to draw connections between their strategies and the strategies being shared with the class in order to deepen and advance their understanding of the mathematical concepts. Further opportunities (and expectations) for students to articulate their thinking and to be held accountable for listening to one another may increase both their expressive and their receptive communication abilities. In other words, being held accountable to explain their thinking and make sense of their peers' thinking will give them practice explaining their own thinking, listening to other students' ways of thinking, and drawing conceptual connections among different strategies and representations.

What's Next?

Stories of teachers engaging in collaborative inquiry focused on using student thinking to inform instructional decisions

What's Next? is a collection of stories documenting professional development experiences shared by elementary teachers working collaboratively to study the complex process of teaching and learning mathematics. Each story in the collection describes practicing teachers studying the thinking processes of real students and using what they learn about those students to make decisions and try to help advance those students' understanding on that day.

The teachers in each story start by learning about how individual students are solving a set of mathematics problems. They use this freshly gathered knowledge of student thinking to develop near-term learning goals for students and a lesson plan tailored to specific students on that specific day. One of the teachers implements the planned lesson while the other teachers observe in real time. The teachers then gather to discuss and reflect on their observations and insights.

In these lessons, the practice of teaching is slowed way down. The stories tell of teachers who are studying student thinking and using that information to plan and implement instructional decisions at a pace that is much slower than it occurs in daily practice. The stories in this collection also depict many aspects in common with formative assessment and lesson study, both of which are a process and not an outcome.

The stories depict real situations that occurred in real time and include both successes and shortcomings. We hope that the stories may be studied and discussed by interested educators so that the lessons and ideas experiences of these teachers and instructional coaches may contribute to additional learning and sharing among other interested teachers.

Learn more about these and other stories at **http://www.teachingisproblemsolving.org/**