Avery Explains That You Cannot Take Eight from Seven on a Subtraction Word Problem

This story is a part of the series:

What's Next? **Stories of Teachers Engaging in Collaborative Inquiry Focused on Using Student Thinking to Inform Instructional Decisions**

What's Next?

Stories of teachers engaging in collaborative inquiry focused on using student thinking to inform instructional decisions

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The research and development reported here were supported by the Florida Department of Education through the U.S. Department of Education's Math-Science Partnership program (grant award #s 371-2355B-5C001, 371-2356B-6C001, 371-2357B-7C004) to Florida State University. The opinions expressed are those of the authors and do not represent views of the Florida Department of Education or the U.S. Department of Education.

Suggested citation: Schoen, R. C. & Champagne, Z. (Eds.) (2017). Avery explains that you cannot take eight from seven on a subtraction word problem. In *What's Next? Stories of teachers engaging in collaborative inquiry focused on using student thinking to inform instructional decisions*. Retrieved from http://www.teachingisproblemsolving.org

Introduction

A group of teachers were studying how to help students learn how to perform multidigit subtraction. In the lesson, the teacher used a subtraction word problem with multidigit values to challenge students to use advanced counting and invented algorithm strategies. During the lesson, a very interesting conversation occurred that challenged the students', as well as the teacher's, existing beliefs about subtraction.

Relevant Florida Mathematics Standards

MAFS.2.OA.1.1 Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

MAFS.2.NBT.2.7 Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.

Background Information

Consider reading chapter seven of *Children's Mathematics: Cognitively Guided Instruction* (Carpenter et al., 2015). This chapter provides background on strategies students use to solve multidigit addition and subtraction problems.

Another source to read that is relevant to the unplanned discussion that occurs during the enactment of the lesson is in the article "Relational Thinking: What's the Difference." In that article, the authors discuss two different ways of thinking about subtraction: subtraction as take away, and subtraction as distance (or difference).

Carpenter, T. P., Fennema, E., Franke, M. L., Levi, L., & Empson, S. B. (2015). *Children's Mathematics: Cognitively Guided Instruction (2nd ed.)*. Portsmouth, NH: Heinemann.

Whitacre, I., Schoen, R. C., Champagne, Z. M., & Goddard, A. (2016). Relational thinking: What's the difference? *Teaching Children Mathematics, 23*(5), 303–308.

Analyzing Student Thinking

This group of teachers developed a subtraction (separate result unknown) problem to pose to a group of second-grade students:

> *Susie had 107 cubes. She gave 78 of those cubes to her best friend. How many cubes did Susie have left?*

To implement this formative assessment item, the teacher read the problem aloud to the whole group. The students were also presented with the problem on paper with plenty of room to show their work. They were asked to solve the problem in whatever way made the most sense to them. The students were asked to record their thinking on the paper so that some other person could understand how they solved the problem without a need for the student to explain further. While the students worked, they had access to manipulatives (including base-ten blocks and snap cubes) and writing materials. As they worked on

During the discussion, it became clear that these second-grade students were correct. You can't take away eight from seven.

If subtraction is conceptualized only as take away, and you have seven blocks, you cannot take eight blocks away.

the problem individually, the teacher circulated and asked students questions to clarify her understanding of their strategies or to assist students when they appeared to be confused or mistaken.

After the students finished, the teacher collected the papers, and the teachers gathered to analyze the students' work and sort them into the following categories based on the strategies they used.

Multidigit Computation Strategies [1]

The student who uses a *direct modeling with ones* strategy represents each multidigit number in the problem as a set of ones using manipulatives or pictures and then counts the objects or pictures to determine the answer. In the example problem, the student could create a set of 107 individual cubes, remove 78 cubes from the set, and count the remaining cubes by ones to determine how many cubes remained.

The student who uses *counting by ones* solves the problem without representing all numbers in the problem. Fingers, objects, or tally marks are often used to keep track of the number of counts. In the example problem, the student might count backwards 78 times from 107 to determine how many cubes were left.

The group of teachers did not anticipate that many students would use these first two strategies, but they thought they should be prepared in case some students did use them.

The student who uses a *direct modeling with tens* strategy represents each multidigit number in the problem using manipulatives or pictures that reflect the base-ten structure of our number system (e.g., with base-ten blocks or base-ten pictures). Then, the student counts the objects or pictures to determine the answer. In the example problem, the student might represent 107 as one 100-block and seven unit blocks or as ten rods of ten and seven ones. The student would then remove seven tens and eight ones from the set of 107. The student would count the remaining cubes to find the answer.

The student who uses an *incrementing* strategy determines the answer by increasing or decreasing partial sums or differences. In the example problem, the student might first decompose 78 into 7, one, and 70, then subtract seven from 107 and have 100 remaining. They student may then subtract one more from 100 to have 99, and then subtract the remaining 70 to find the answer.

The student who uses a *combining the same units* strategy operates on the tens and ones separately and then combines the partial sums to get a final result. In the example problem,

1 *The descriptions of strategies presented in this section are the current descriptions used by our team, and we consider them to be fluid, as our understanding of these ideas continues to evolve. For a more detailed discussion of these terms, consider reading Carpenter et al. (2015).*

Direct modeling with ones	*Direct modeling with tens*	*Counting by ones*	*Combining same units*	*Incrementing*	*Standard algorithm*	*Other*
Shantral	Emma	Cyndi			Steven (incorrect)	Ollie
Eva	Adaline				Bradley	
	Avery				Stella (incorrect)	
					Quinn	
					Michael (incorrect)	
					Matt	
					Mavis	
					Erikah	
					Donald (incorrect)	

Figure 1. The classification of student strategies on the cubes problem.

the student might subtract 70 from 100 to get 30 and then eight from seven. The student may then know that one more still needed to be subtracted and would answer 29. The teachers understood that this strategy represented very complex thinking but again wanted to be prepared should such a strategy be encountered in student work.

The student who uses the *U.S. standard algorithm* begins by writing the problem vertically with the 107 on the top, writing 78 below it, and drawing a horizontal line below the 78. Directing their attention to the ones place, the student notices that 7 is less than 8 and proceeds to regroup numbers so that the 1 in the 100 becomes ten tens, and is moved to the tens column. One of the tens is combined with the 7 to make 17, so 9 tens remain in the tens column. The student evaluates 17 – 8 and writes the difference below the line under the 8. The student then evaluates 9 – 7 and writes the difference (i.e., 2) below the line under the 7, forming 29.

The student who attempts a strategy that does not clearly fit into one of the strategies listed above is placed in the *other* category.

Figure 1 shows the teachers' classification of the students' strategies on the cubes problem.

The teachers were surprised at the predominance of the *standard algorithm* strategy. The teachers also noticed that a large number of students in the standard algorithm category did not execute the strategy correctly. Many of them were "subtracting up" to get answers such as 171 or 31.

On the basis of this information, the teachers developed the following goal:

> *Encourage students to make sense of the strategies they and other students use.*

They recognized that this goal would probably not be reached in one afternoon. It might be a long-term goal that would take time, but they moved forward with planning an instructional sequence to help move students toward this goal.

Planning for the Lesson

The group decided to plan a lesson that would engage the class in a whole-group discussion of the strategies students used previously rather than posing a new problem. Because the goal was to help students use strategies they understood and could explain, the teachers decided to spend the time working with the strategies students had used correctly. They were also fairly certain that, if

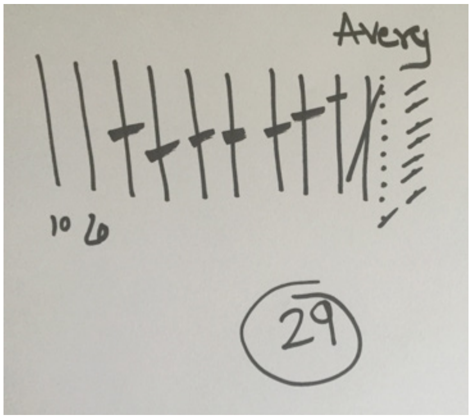

Figure 2: Avery's work showing a ***direct modeling with tens*** *strategy.*

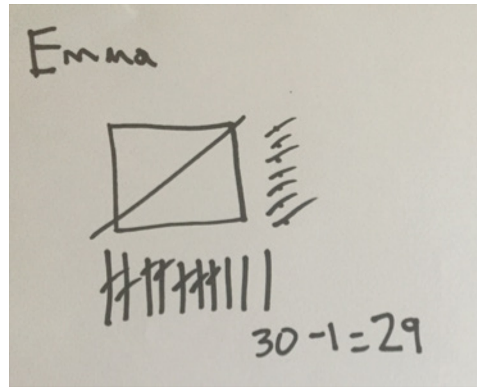

Figure 3. Emma's work showing a ***direct modeling with tens*** *strategy.*

the students were given a new separate result unknown problem, they would use the same strategies and make the same errors they had made previously.

The group decided to share Emma and Avery's work as a basis for discussion about other ways to solve multidigit subtraction problems. Notice that Avery (Figure 2) first drew a picture representing ten tens and seven ones. He then crossed out the seven ones, changed a ten into ten ones, and crossed out one more of those. He then crossed out seven tens and counted by tens and ones to find that 29 cubes were left.

Notice that Emma (Figure 3) first drew a 100 block and seven ones. She then changed the 100 block into ten tens. She then crossed out the seven ones and seven tens. That left 30. She knew she needed to subtract one more, because she knew she had only subtracted 77. She arrived at an answer of 29 cubes as well.

The teachers agreed that, although the teacher would be sharing the work of just two students during the lesson, a concerted effort would be made to include as many students as possible during the discussion. They planned to implement a number of turn-and-talk segments to encourage all students to participate in making sense of Avery and Emma's strategies.

The teachers also discussed the possibility of posing a correct *standard algorithm* strategy along with Emma and Avery's work and then asking the students to explain which of the three strategies were most similar and why. The teachers decided that this question had two valid answers. Emma's and Avery's strategies are similar, because they both used drawings based on place value. Avery's and the *standard algorithm* are also similar, because they used regrouping strategies and worked on tens and ones separately.

Lesson Plan

In the ***Analyzing Student Thinking*** section, the teachers developed the following goal:

> *Encourage students to make sense of the strategies they and other students use.*

First ask the student you have selected to explain his or her strategy to the class. Then have the students turn and talk to each other to discuss the strategy described. Provide some specific guiding questions for each strategy; plan the opening questions in advance. Finally, lead a closing discussion about the strategy, eliciting responses from a variety of students in the class. Repeat this process two times for each piece of student work chosen to be shared (see the ***Planning for the Lesson*** section for the examples in this lesson).

1. Review the problem the students completed earlier in the day.

2. Call on Avery and ask him to bring his work

to the document camera and explain his strategy to the class. Ask questions to help the students notice important features of Avery's strategy, including the trading of one ten for ten ones.

3. Ask the students to turn and talk with their neighbors about Avery's strategy. Provide guiding questions as necessary to help point out any features in the work you'd like to connect later. For example, you might elucidate how he represented 107 and what he decided to do when he didn't have any more ones to take away.

4. Call the students back together and have them share what they noticed about Avery's work. Consider asking:

   a. What did Avery do to solve the problem?
   b. What does Avery's picture represent?
   c. How does Avery's strategy help you make sense of the problem?
   d. Is Avery's strategy similar to any other strategies we have seen?

5. Call Emma to the front of the class to share her work.

6. Ask students if they can explain how Emma knew to take one more away and why she didn't represent it with her drawings.

7. Ask the students to turn and talk with their neighbors about Emma's strategy. Provide guiding questions as necessary to help point out any features in her thinking or representation you'd like to connect later. For example, you might ask the class to restate why Emma subtracted one from 30.

8. Call the group back together and have students share what they noticed about Emma's work. Encourage students to attend to details in her strategy and compare and contrast it with Avery's strategy.

9. Repeat the process with Bradley showing how he used the *standard algorithm* to subtract.

The teachers thought it was important to expose students to different ways of thinking about the subtraction operation. They decided that focusing on subtraction as take away could be impeding their students' future understanding of the operation.

10. Hold a discussion about how the three strategies are alike and how they are different. Consider demonstrating other ways to represent Avery and Emma's strategy without using pictures. For example, you might choose to represent each step of Avery or Emma's work with equations and expressions to show how to represent the steps they took to solve the problem.

Reflection

This lesson was intended to focus on representations that were not the standard algorithm. The lesson did accomplish some of that, but the highlight came from an unplanned discussion about Avery's strategy.

As Avery was showing his solution to the problem, the teachers started to notice that although he used a drawing to solve the problem, he was using some of the language typically used in the standard algorithm. In the middle of his explanation, he said, "you can't take eight from seven, so

I have to borrow."

After Avery finished describing his strategy, the teacher asked the class, "Avery said that you can't take eight from seven. Do you all agree with that statement?" She paused as almost all of the hands went up. She then rephrased the question and asked, "how many of you agree that you can't subtract eight from seven?"

This time about 60% of the class raised their hands. The teacher asked, "Some of you don't agree. How do you feel about the idea that you can't subtract eight from seven?"

One student responded with, "I feel sad."

Keeping the momentum going, the teacher asked the students who did agree with the statement to explain their thinking. She noticed that Cyndi had her hand raised and so the teacher called on her. Cyndi said, "you can't, because then it would be negative one. So you have to regroup."

Then Matt eagerly raised his hand. He clearly had something he really wanted to say. The teacher called on Matt, and he said, "If you subtract eight from seven, you would get negative one. But, we are in second grade, so we can't do that. If we were in a higher grade, we could, but we are in second grade, so we can't."

The group of teachers enjoyed hearing from Matt and, as expected, Matt's comments were something that the group of teachers had a lengthy conversation about during their final discussion.

During the discussion, it became clear that these second-grade students were correct. You can't take away eight from seven. If subtraction is conceptualized only as take away, and you have seven blocks, you cannot take eight blocks away. Of course, you can subtract eight from seven, but you cannot take eight away from seven. Subtraction is more abstract than taking away physical objects.

In this scenario, the teacher changed her question to, "can you subtract eight from seven." The teachers came to agree that the terms subtract and take away are not synonymous and these students convinced the group that although you can subtract eight from seven, you cannot really take eight away from seven.

The teachers agreed that more lessons would be needed to attain their original goal of encouraging students to use strategies they understand when solving separate result unknown problems, but a secondary goal emerged as well. The teachers thought it was important to expose students to different ways of thinking about the subtraction operation. They decided that focusing on subtraction as take away could be impeding their students' future understanding of the operation. The teachers set personal goals to teach lessons that provided more opportunities to conceptualize subtraction as distance, or difference, between two numbers on the number line or numbers of objects rather than just take away. Several teachers acknowledged that they use the words "take away" when they read the subtract symbol, and they decided to try to avoid doing so in the future.

What's Next?

Stories of teachers engaging in collaborative inquiry focused on using student thinking to inform instructional decisions

What's Next? is a collection of stories documenting professional development experiences shared by elementary teachers working collaboratively to study the complex process of teaching and learning mathematics. Each story in the collection describes practicing teachers studying the thinking processes of real students and using what they learn about those students to make decisions and try to help advance those students' understanding on that day.

The teachers in the stories start by learning about how individual students solve a set of mathematics problems. They use this freshly gathered knowledge of student thinking to develop near-term learning goals for students and a lesson plan tailored to specific students on that specific day. One of the teachers implements the planned lesson while the others observe in real time. The teachers then gather to discuss and reflect on their observations and insights.

In these lessons, the practice of teaching is slowed way down. The stories tell of teachers who are studying student thinking and using that information to plan and implement instructional decisions at a pace that is much slower than is typical in daily practice. They depict a process with many aspects in common with formative assessment and lesson study, both of which are also conceptualized as processes and not outcomes.

The stories depict real situations that occurred in real time and include both successes and shortcomings. We hope they will be studied and discussed by interested educators so that the lessons and ideas experienced by these teachers and instructional coaches will contribute to additional learning and sharing among other interested teachers.

Learn more about these and other stories at **http://www.teachingisproblemsolving.org/**