Students Making Sense of Equal-Sharing Problems

This story is a part of the series:

***What's Next?* Stories of Teachers Engaging in Collaborative Inquiry Focused on Using Student Thinking to Inform Instructional Decisions**

What's Next?

Stories of teachers engaging in collaborative inquiry focused on using student thinking to inform instructional decisions

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Introduction

Teachers developing a lesson on unit fractions with third-grade students began with a formative assessment task intended to gauge the students' understanding of unit fractions. They then developed a lesson that would review student solutions, pose a new problem, and share student solutions to the new problem.

Relevant Florida Mathematics Standards

MAFS.3.NF.1.1 Understand a fraction 1/b as the quantity formed by 1 part when a whole is portioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size 1/b.

Background Information

Consider reading chapter one in Extending Children's Mathematics: Fractions and Decimals (Empson & Levi, 2011). This chapter provides a more detailed explanation of equal sharing problems and the strategies that children typically use when solving them.

Empson, S. & Levi, L. (2011). *Extending Children's Mathematics: Fractions and Decimals.* Portsmouth, NH: Heinemann.

Analyzing Student Thinking

The following two problems were posed to a classroom of third-grade students in early January. The students had access to writing materials and were asked to solve the problem in whatever way made the most sense to them. The students completed the problems individually at their desks and the teacher circulated around the room during this time. The problems were selected because the teachers wanted to learn how these students would solve these types of problems and how they were thinking about fractional quantities before the instruction unit on fractions was implemented in their classroom.

Problem A: *If 4 people share 5 cookies equally, how much cookie would each person get?*

Problem B: *If 4 people share 7 brownies equally, how much brownie would each person get?*

Categorization of Students' Problem Solving Strategies

As the students worked, the teachers considered the following categories as general groups into which to sort the students' work[1].

The student who uses a *non-anticipatory sharing* strategy does not think in advance about the number of sharers or the amount to be shared, does not attempt to solve the problem with a plan, and relies on strategies such as repeated halving or trial and error. In problem A, the student would divide each of the five cookies in half and pass out a piece to each of the four people. The student would then take the last piece and break it into three pieces so that in the end all four people have three pieces. See Figure 1 for an example of student work created by a student who used this type of strategy. Note that this type of thinking does not usually lead to equivalent-sized pieces for each person.

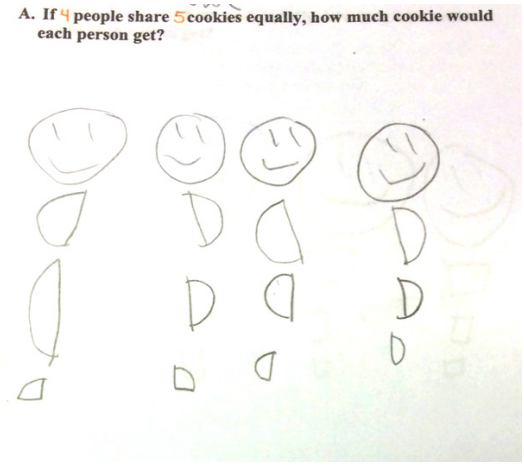

Figure 1. An example of a student's ***non-anticipatory sharing*** *strategy.*

1 The descriptions of strategies presented in this section are the current descriptions used by our team, and we consider them to be fluid, as our understanding of these ideas continues to evolve. For a more detailed discussion of these terms consider reading Empson, S., & Levi, L. (2011).

The problems were selected because the teachers wanted to learn how these students would solve these types of problems and how they were thinking about fractional quantities before the instruction unit on fractions was implemented in their classroom.

The student who uses an *additive coordination-sharing one item at a time* strategy starts by splitting the first item to be shared in exactly as many parts as people who will receive shares, then repeat the process for each item until everything has been shared equally. In problem A, this strategy may involve a student taking one of the cookies, breaking it into four pieces, and distributing them to each of the four people. The student would continue to do the same for each of the five cookies until each person has five cookie quarters. See Figure 2 for an example of student work created by a student who used this type of strategy.

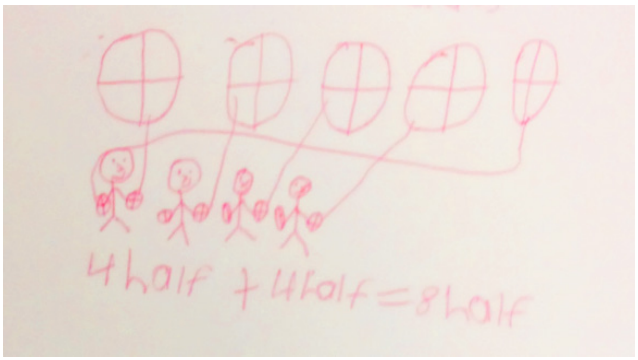

Figure 2. An example of a student's ***additive coordination - sharing one item at a time*** *strategy.*

The student who uses an *additive coordination-sharing groups of items* strategy shares out the wholes equally to the group until not enough wholes remain and then partitions the remaining wholes to provide equal shares. In problem A, the student would give each student one cookie and then split the last cookie into four equal-sized pieces. See Figure 3 for an example of student work created by a student who used this type of strategy.

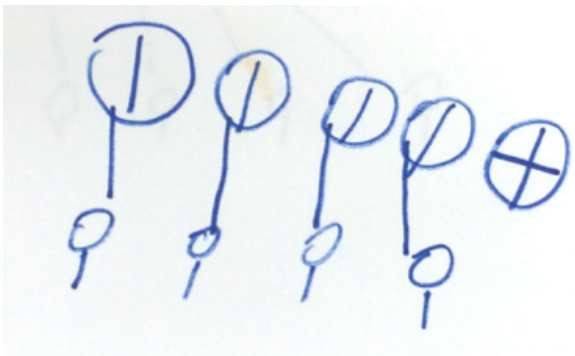

Figure 3. An example of a student's ***additive coordination - sharing groups of items*** *strategy.*

Non-Anticipatory Strategies		Anticipatory Strategies		
Nonvalid	Non-Anticipatory	Sharing one item at a time	Sharing groups of items	Multiplicative coordination
Jaylin	Katie	Roxella	Kristopher Jason Kaylee Amber Brandon Corry	Stephan

Figure 4. Students' names, sorted according to the type of strategy they used to solve Problem A.

The student who uses a *multiplicative coordination* strategy sees the operation of division as the outcome written as a fraction. In other words, the student sees the quotient as a fraction. In problem A, the student would say the answer is 5/4, because four people are sharing five cookies.

After the students solved the problem, the teachers sorted the student's strategies into these categories, as depicted in Figure 4.

The teachers noticed an interesting distribution of student strategies—many students already used *anticipatory* strategies—but the most pressing observation shared during this discussion was incidental. The teachers observed that less than half of the class knew how to name and write unit fractions correctly. On the basis of this information, the teachers developed the following goals for the lesson:

> *1. The students should come to understand that unit fractions can be created by dividing a whole into two or more equal-sized pieces.*
>
> *2. The students should come to understand how to name and write unit fractions.*

Planning for the Lesson

The teachers decided to have Katie, Jaylin, Corry, Roxella, Stephan, and Kaylee share their thinking on the previous problems with the class, so that the students would see how others solved these types of problems. A secondary goal of choosing these students was that they included a variety of different solutions, including *non-anticipatory* and *anticipatory* strategies.

The teachers also worked to develop a problem for students to solve that would illuminate and advance their thinking toward understanding that unit fractions could be created by dividing a whole into equal-sized pieces. The teachers eventually settled on the following problem:

> *Three people want to share 5 cookies equally. If they share all of the cookies, how many cookies would each person get?*

The problem is one of equal sharing, and the teachers thought it would allow students the opportunity to anticipate how they might share the cookies in advance of doing the cutting. The numbers in the problem are sufficient for students to anticipate giving each person one whole cookie and then having to share the remaining two cookies among the three people, providing the opportunity to engage with unit fractions. To address their second goal, the teachers also planned to demonstrate and explain conventional ways unit fractions are named and written in mathematics.

Lesson Plan

The following lesson plan was implemented after the initial information was collected on the students. In the planning for this lesson, the teachers established the following learning goals:

> *1. The students should come to understand that unit fractions can be created by dividing a whole into two or more equal-sized pieces.*
>
> *2. The students should come to understand how to name and write unit fractions.*

1. In preparation for the lesson, identify student work—selected for the purpose of having the student share their work with the class—that demonstrates anticipatory thinking and clearly communicates the student's strategy. Next, carefully arrange two to three such student responses in a useful order. One useful order would be to sequence the strategies from least sophisticated to most sophisticated. Another could be to group student work that uses similar models or strategies with key differences (to draw attention to those different ways of expressing ideas).

2. Begin the lesson by having students who used an anticipatory strategy share the work they did on the earlier problem, and have these students share the paths they took in arriving at the solution. Doing so allows the students the opportunity to articulate their thinking on the problem and the other students to interact with that idea.

3. After the selected student work has been shared, pose the new problem and briefly explain to the students that, for this problem, they should be careful record their thinking so that someone else can clearly see how they arrived at their answers. If your students are not used to recording their thinking, explain that some appropriate ways of expressing their ideas could be through pictures, numbers, words, or some combination of those.

> New Problem: *Three people want to share 5 cookies equally. If they share all of the cookies, how many cookies would each person get?*

4. Be sure that students have access to writing materials while they work. Some students may also find linking cubes helpful, so consider having these available, but do not require students to use them.

5. While the students work, circulate and observe to ensure that students are using *anticipatory* strategies similar to the ones shared in the Analyzing Student Thinking section. When you find students who are doing so, ask whether they would be willing to share their thinking during the sharing portion of the lesson. Those students can therefore begin thinking about what they would like to say about their work.

6. When you find students using non-anticipatory strategies or struggling to solve the problem correctly, consider trying one of the following strategies:

a. Remind the student to think about one of the strategies shared during the opening and ask whether they would consider trying one of those ways.

b. Ask the student a more general question such as, "Is there another way you could think about this problem" or "Are there other things to consider before you start splitting up the cheese slices?"

7. When you find students who have correctly finished the task earlier than the other students, consider trying one of the following strategies:

a. Provide the more challenging problem listed in the differentiation section below.

b. Ask the student to consider other ways to represent the answer.

c. Ask the student to consider how the number

of cheese slices that each student gets (5/3) is related to the number of cheese slices shared and the number of people sharing. This can help the students move toward using a multiplicative strategy to solve these problems.

8. Invite three to four students. These students should have used an *anticipatory* strategy to share their thinking with the class. Select students deliberately so that they will, collectively, demonstrate a variety of related strategies, including those that use pictures, numbers, words, or a combination of representations. Try to select strategies that have some commonality but differ in key ways.

a. As students share, be sure to remember to name the unit fractions according to conventional nomenclature so that students begin to gain familiarity with the conventions for naming and writing unit fractions.

Reflection

The teacher began the lesson by having six students share their thinking with the class. This process took approximately 30 minutes, much longer than the teacher expected. The time available for the students to work and share their thinking on the new problem was therefore limited. The teachers agreed that, in future lessons, the initial sharing of student strategies should be limited to 15 minutes. In retrospect, they decided it may have been a better plan to select no more than three students to share their strategies with the class.

The teachers observed that more students used a *non-anticipatory* strategy than expected, based on the strategies they used in the problem given before the lesson. A possible reason the teachers cited was that the numbers were more difficult in this problem, because the cookie was divided into thirds, and that some students may not have been as familiar with these combinations as they were with halves, fourths, or other powers of two. The teachers recognized that not all students can be expected to understand this idea completely after just one lesson. These ideas were new to the students, and comprehension takes time to develop. They made a note to be sure to pose problems fractions involving thirds, fifths, and sevenths so that students could develop their understanding of these types of fractional parts along with their understanding of halves, fourths, and eighths.

The teacher introduced conventions of language and notation to name and write unit fractions, and students seemed to become more familiar with using these conventions over the course of the lesson. The teachers anticipate that comprehension of these conventions will continue to develop over time. The teachers agreed that they must be very deliberate about how and when they introduce and reinforce the formal conventions for mathematical notation.

The teacher in this classroom anticipates continuing to use equal-sharing problems to encourage the development of all of these ideas with the idea that students will create and learn more advanced strategies for solving such problems from one another as their ideas become more abstract and sophisticated.

The teachers recognized that not all students can be expected to understand this idea completely after just one lesson.

What's Next?

Stories of teachers engaging in collaborative inquiry focused on using student thinking to inform instructional decisions

What's Next? is a collection of stories documenting professional development experiences shared by elementary teachers working collaboratively to study the complex process of teaching and learning mathematics. Each story in the collection describes practicing teachers studying the thinking processes of real students and using what they learn about those students to make decisions and try to help advance those students' understanding on that day.

The teachers in each story start by learning about how individual students are solving a set of mathematics problems. They use this freshly gathered knowledge of student thinking to develop near-term learning goals for students and a lesson plan tailored to specific students on that specific day. One of the teachers implements the planned lesson while the other teachers observe in real time. The teachers then gather to discuss and reflect on their observations and insights.

In these lessons, the practice of teaching is slowed way down. The stories tell of teachers who are studying student thinking and using that information to plan and implement instructional decisions at a pace that is much slower than it occurs in daily practice. The stories in this collection also depict many aspects in common with formative assessment and lesson study, both of which are a process and not an outcome.

The stories depict real situations that occurred in real time and include both successes and shortcomings. We hope that the stories may be studied and discussed by interested educators so that the lessons and ideas experiences of these teachers and instructional coaches may contribute to additional learning and sharing among other interested teachers.

Learn more about these and other stories at **http://www.teachingisproblemsolving.org/**