Jessiah Uses Repeated Halving Strategies on Equal-Sharing Problems

This story is a part of the series:

***What's Next?* Stories of Teachers Engaging in Collaborative Inquiry Focused on Using Student Thinking to Inform Instructional Decisions**

What's Next?

Stories of teachers engaging in collaborative inquiry focused on using student thinking to inform instructional decisions

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Introduction

A teacher took the following path in developing a mathematics lesson involving equal-sharing problems with fractional amounts for her fifth-grade students. The lesson begins with a formative-assessment task that the teacher used to gauge her students' understanding of this topic. The teacher analyzed the student work and then developed a near-term learning goal based on their current understanding. The teacher ultimately developed a brief lesson that would review student solutions, pose a new problem, and engage students in discussion about their solutions to the new problem.

Relevant Florida Mathematics Standards

MAFS.5.NF.3 Interpret a fraction as division of the numerator by the denominator (a/b = a ÷ b). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

Background Information

The interested reader may want to read chapter one from *Extending Children's Mathematics* (Empson & Levi, 2011). The chapter provides explanation and discussion of equal-sharing problems and the strategies that children typically use when solving them.

Carpenter, T. P, Fennema, E., Franke, M. L., Levi, L., & Empson, S. B. (2015). *Children's Mathematics: Cognitively Guided Instruction*. Portsmouth, NH: Heinemann.

Empson, S. & Levi, L. (2011). *Extending Children's Mathematics: Fractions and Decimals.* Portsmouth, NH: Heinemann.

Analyzing Student Thinking

The following problem was posed to a classroom of fifth-grade students in late September. The students had access to writing materials (i.e., pencils, paper) and were asked to solve the problem in whatever way made sense to them. At this point in the year, the students had not yet done formal work with fractions. The intent of the problem and the timing was to determine how the students would solve this problem before any formal instruction.

> *There were 4 hungry people that are sharing 11 brownies. How much brownie should each person get if they want everyone to have the same amount?*

As the students worked, the teacher considered the following categories as general groups in which the students might be placed.[1]

Types of Strategies and Related Terminology

A student using a *nonvalid* strategy would not arrive at a correct answer even when enacting the strategy correctly, for example, each person two brownies and saying that three brownies were left over.

A student using a *non-anticipatory-sharing* strategy does not think in advance about the number of sharers or the amount to be shared. Figure 1 contains one drawing made by a student who used a *non-anticipatory-sharing strategy*. He gave each person two brownies and then cut each of the remaining three brownies in half. The student then gives a half to each person and decides not to distribute the remaining two halves (Figure 1).

There are different variations of *non-anticipatory-sharing* strategies that students may use to solve equal sharing problems. This is only one example.

1 The descriptions of strategies presented in this section are the current descriptions used by our team, and we consider them to be fluid, as our understanding of these ideas continues to evolve. To learn more about these terms, consider reading Extending Children's Mathematics (Empson, & Levi, 2011).

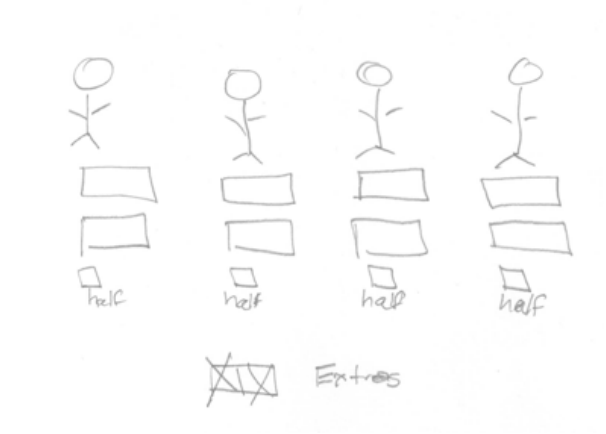

Figure 1. An example of a ***non-anticipatory*** *sharing strategy*

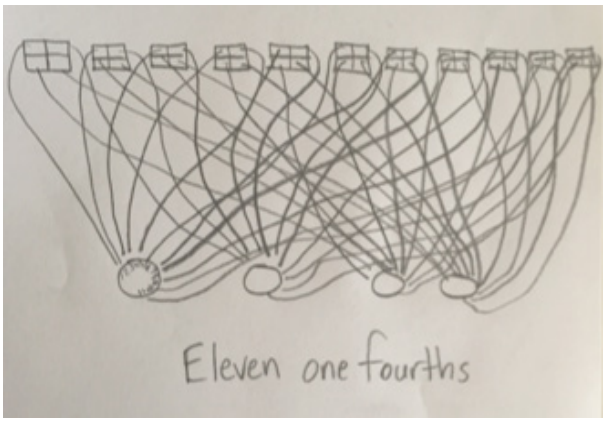

Figure 2. The strategy of a student who shared one item at a time distribute 11 brownies equally among four people.

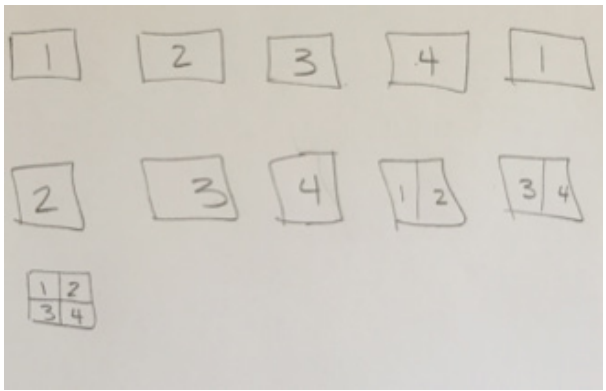

Figure 3. The strategy of a student who shared groups of items to distribute 11 brownies among four people.

It is also important to note that *non-anticipatory-sharing* strategies do not always produce an incorrect solution. With the numbers in this problem (i.e., four sharing 11), halving each extra brownie after sharing the wholes (a typical strategy students use), results in an incorrect solution.

Additive Coordination Strategies

The student using a *sharing one item at a time* strategy uses a model to show that each object being shared is partitioned into the number of parts that matches the sharers. In this example, the student represents each of the 11 brownies. Then splits each brownie into fourths because four people are sharing. The student then gives one-fourth of each brownie to each of the 11 people (Figure 2).

A student using a *sharing groups of items* strategy shares out the wholes equally to the group until not enough wholes remain to continue and then partitions the remaining wholes to provide equal shares. In the example of this strategy provided in Figure 3, the student first distributes two whole brownies to each person, cuts two of the remaining three in half to giving each person one half of a brownie, and finally cuts the remaining brownie into fourths and gives each person one-fourth.

Multiplicative Coordination

A student using a *multiplicative coordination* strategy sees the fraction as the quotient. The student may write something like, "The answer is eleven-fourths, because four people are sharing 11 brownies."

After the students in this classroom solved the problem, the teacher sorted each student's work into strategy categories. Figure 4 represents the strategies used by students in the class to solve the problem.

Non-Anticipatory Strategies		Anticipatory Strategies		
Nonvalid	Non-Anticipatory	Sharing One Item at a Time	Sharing Groups of Items	Multiplicative Coordination
Athey	Kori	Jessiah		
Kinleigh	Aiden	Savannah		
	Gavin	Avery		
	Lauren	Kiadira		
	Kellen	Caleb		
	Ryan	Fisher		
	Ethan	Cash		
		Jameson		
		Alyssa		

Figure 4. A snapshot of the strategies used by the students in the class

Strategies Used by Students in This Class

Learning Goal for This Group of Students on This Day

The teacher noticed that about half of the students in the class were using a *non-anticipatory* strategy, and those students who were using anticipatory strategies were *sharing one item at a time*. On the basis of this information, the teacher developed the following primary learning goal:

> *Move students from non-anticipatory to anticipatory sharing strategies (other than halving).*

This primary learning goal was developed to focus on the students who used *non-anticipatory* thinking, but the teacher also planned to look for students who shared one item at a time in the previous problem to begin *sharing groups of items*.

Planning for the Lesson

The teacher planned, in the next lesson, to have Jessiah, Avery, and Alyssa, all of whom used strategies involving sharing the brownies one item at a time, share with the class their thinking on the problem. They were chosen, because their strategies would provide an opportunity for students in the *non-anticipatory* group to be exposed to strategies based on anticipatory thinking and to learn how anticipatory thinking may be communicated in words and symbols. The teacher planned to encourage students who previously used *non-anticipatory* strategies to consider using one of the anticipatory strategies shared by their peers when working on the next problem. This technique helps students to be more aware of the goal for the lesson and allows students to experiment with the new strategies that were shared.

The teacher strove to develop a problem for students to solve that would illuminate and advance their thinking toward using anticipatory thinking. The teacher eventually settled on the following problem:

> *There were 3 children at a party. There were 4 candy bars at the party. They want to share the candy bars so that every child has the same amount with no candy bars left over. How many candy bars will each child have?*

This problem is an example of an equal-sharing problem. The teacher thought the numbers chosen for this problem would allow students the opportunity to anticipate how they might share the candy bars in advance of doing the cutting. The numbers are also suitable for students to anticipate giving everyone one whole candy bar and then only having one candy bar left to split among the three children. Because only one item is left after the whole candy bars are shared, the problem is much easier than dividing the three brownies that were left in the previous problem among four people, and this might make it easier for students to begin to use anticipatory strategies.

Differentiation strategies

Planning to encounter students who may not initially be successful with solving the problem, the teacher planned to ask them to solve a problem in which the division leaves no remainder. She prepared the following problem in case this occurred.

> *Three children were at a party. They had six candy bars. They want to share all of the candy bars so that everyone has the same amount. How many candy bars can each child have?*

The teacher planned to encourage students who previously used ***non-anticipatory*** *strategies to consider using one of the anticipatory strategies shared by their peers when working on the next problem. This technique helps students to be more aware of the goal for the lesson and allows students to experiment with the new strategies that were shared.*

If the students were able to solve that problem, the teacher would pose the original problem again and ask them, "What are some things we could do with that left-over candy bar?"

For students who finished quickly and used anticipatory thinking, the teacher planned to challenge them with a similar problem in which more than one candy bar was left over to determine whether they could then use anticipatory thinking on that problem. She prepared the following problem to pose to these students.

> *Five children were at a party. They had eight candy bars. They want to share the candy bars so that everyone has the same amount. How many candy bars each child everyone have?*

While the students worked, the teacher watched for students who used anticipatory thinking by partitioning the left-over candy bar into three pieces before distributing it. The teacher also intended to make note of these students to ensure that they could share their thinking in the summarizing section of the lesson plan.

Lesson Plan

The following lesson plan was implemented after the initial information was collected on the students through Phase 2, and the following primary learning goal was developed:

Move students from non-anticipatory to anticipatory sharing strategies (other than halving).

1. In preparation for the lesson, locate student work that demonstrates anticipatory thinking and clearly communicates the student's thinking. The examples in the additive coordination sections are good examples. Next, carefully sequence the two or three student responses that the students will share. One possible sequence—often the best—would be from least sophisticated to most sophisticated, but another possibility would be to present solutions that use similar models or strategies with small differences in representations or explanations.

2. Begin the lesson by having the students share the responses you chose from the prior problem and have each student share his or her path to arriving at the solution. This opportunity provides opportunities for the sharing students to practice communicating their thinking on the problem and for the other students to interact with those student's ideas.

3. Pose the problem. *There were 5 children at a party. They have 8 candy bars. They want to share the candy bars so that everyone has the same amount. How many candy bars can everyone have?*

4. Briefly discuss the following expectations with the students:

   a. Communicate to the students that they should record their thinking so that someone else can clearly see how they arrived at their answers.

   b. Explain that they are expected to communicate this thinking with pictures, numbers, words, or some combination.

5. Be sure that students have access to writing materials while they work. Linking cubes may also be beneficial for some students. Consider having these available in a nearby location but do not require their use. While the students work, circulate to watch for students using anticipatory strategies like the ones shared above.

6. When you find such students, ask a few of them whether they would be willing to share their thinking during the summarizing portion of the lesson. Doing so allows those students to begin thinking about what they would like to say in reference to their work. You may choose students on the basis of the clarity of their strategies or as a way to include less active students in the discussion.

7. When you find students who are not using anticipatory thinking or are struggling to solve the problem correctly, consider trying one of the following strategies:

   a. Remind the student of one of the strategies shared during the opening and ask him or her to consider trying one of those ways.

   b. Ask the student a more general question such as, "Is there another way you could think about this problem?" or "Are there other things to consider before you start splitting up the candy bars?"

8. When you find students who have correctly finished the task during this phase, consider trying one of the following strategies:

   a. Provide the more challenging problem listed in the differentiation section above.

   b. Ask the student to consider other ways to represent the answer.

   c. Ask the student to consider the amount of a candy bar that each student gets (i.e., 4/3)

and how that amount is related to the number of candy bars shared and the number of people sharing. Doing so can help the student move toward using a multiplicative strategy to solve these problems and to understand that 4/3=4÷3.

9. Invite three to four students to share their thinking. These students should be using anticipatory strategies, and the teacher should encourage them to use pictures, numbers, words, or a combination of these representations to communicate their ideas. Sequencing the presentations carefully may be important. Consider sequencing them from less sophisticated to more sophisticated or grouping by models or strategies used. Leave the representations in view so that students can be encouraged to compare and contrast the various strategies.

Reflection

The teacher began the lesson by having three students share their thinking with the class. This process took approximately 30 minutes, which was much longer than the teacher expected. The time available for the students to work and share their thinking on the new problem was therefore limited. The teacher also realized that some of the students were not paying as close attention to the students sharing as she expected. The teacher noted that, in future lessons, the initial sharing session should be limited to 15 minutes.

An major important observation was that not all students moved on to anticipatory thinking, but the teacher recognized that not all students can be expected to understand this idea completely after one lesson. The ideas were new to the students, and comprehension takes time to develop, but many students were able to make progress toward the goal. Five students who previously used *non-anticipatory* strategies in the first problem did demonstrate anticipatory thinking in solving the problem during the lesson.

The teacher anticipates continuing to use equal-sharing problems to encourage the development of ideas through discussion and to expose students to more advanced strategies for solving such problems.

The teacher also noticed that students were struggling with the correct way to write the fractions and fractional parts that they were communicating. Some students struggled with where and how to record the numerators and denominators, and others struggled correctly representing mixed numbers. The teacher decided to follow up with subsequent lessons on how to use the correct notation for writing fractions.

What's Next?

Stories of teachers engaging in collaborative inquiry focused on using student thinking to inform instructional decisions

What's Next? is a collection of stories documenting professional development experiences shared by elementary teachers working collaboratively to study the complex process of teaching and learning mathematics. Each story in the collection describes practicing teachers studying the thinking processes of real students and using what they learn about those students to make decisions and try to help advance those students' understanding on that day.

The teachers in each story start by learning about how individual students are solving a set of mathematics problems. They use this freshly gathered knowledge of student thinking to develop near-term learning goals for students and a lesson plan tailored to specific students on that specific day. One of the teachers implements the planned lesson while the other teachers observe in real time. The teachers then gather to discuss and reflect on their observations and insights.

In these lessons, the practice of teaching is slowed way down. The stories tell of teachers who are studying student thinking and using that information to plan and implement instructional decisions at a pace that is much slower than it occurs in daily practice. The stories in this collection also depict many aspects in common with formative assessment and lesson study, both of which are a process and not an outcome.

The stories depict real situations that occurred in real time and include both successes and shortcomings. We hope that the stories may be studied and discussed by interested educators so that the lessons and ideas experiences of these teachers and instructional coaches may contribute to additional learning and sharing among other interested teachers.

Learn more about these and other stories at **http://www.teachingisproblemsolving.org/**