Working on Fact Fluency with Addition and the Importance of Derived Fact Strategies

This story is a part of the series:

***What's Next?* Stories of Teachers Engaging in Collaborative Inquiry Focused on Using Student Thinking to Inform Instructional Decisions**

What's Next?

Stories of teachers engaging in collaborative inquiry focused on using student thinking to inform instructional decisions

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Introduction

In the context of a professional-development session, a group of teachers designed a lesson for a first-grade class with the intent to help students move toward greater fluency with addition facts within 20. The teachers first interviewed the students to learn how they solved addition-fact problems. On the basis of their observation that the majority of students used either a *direct modeling* or a *counting* strategy, the teachers designed a lesson with the goal of encouraging the class to begin using strategies based on recall of addition facts.

Relevant Florida Mathematics Standards

MAFS.1.OA.3.6 Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).

Background Information

Consider reading chapter three in *Children's Mathematics: Cognitively Guided Instruction* (Carpenter et al., 2015). This chapter provides background on the varied ways children solve addition and subtraction problems, including the use of *known and derived-fact strategies*. It also expands upon the strategies explained in the section titled Analyzing Student Thinking. Another discussion on fact fluency is contained in the article "Fluency with basic addition" (Kling, 2011), which discusses components of fluency and how they are addressed in curriculum standards for mathematics. For more information on assessing fact fluency, consider reading "Assessing basic fact fluency" (Kling & Bay-Williams, 2014), which provides a variety of ways to assess students' fluency with basic math facts.

Carpenter, T. P., Fennema, E., Franke, M. L., Levi, L., & Empson, S. B. (2015). *Children's Mathematics: Cognitively Guided Instruction (2nd ed.)*. Portsmouth, NH: Heinemann.

Kling, G. (2011). Fluency with basic addition. *Teaching Children Mathematics, 18*(2), 80–88.

Kling, G., & Bay-Williams, J. M. (2014). Assessing basic fact fluency. *Teaching Children Mathematics, 20*(8), 488–497.

Analyzing Student Thinking

A group of teachers conducted one-on-one interviews with each individual student in a classroom of first graders. The interviewers were designed to assess their students' fluency with addition facts. The interviews involved verbally posing a series of addition problems—one at a time—for students to evaluate. Students were asked to evaluate each of the expressions in Figure 1, starting at the top of the left-hand column, proceeding down the column, moving to the top of the right-hand column, and proceeding down the right-hand column. On each item, after the student provided an answer, the interviewer asked the student to explain *how*[1] he or she had arrived at the answer. The interviewers retained the option to skip any problem(s) thought to pose too much of a challenge for any particular student. During the interview, the students had access to markers, paper, and manipulatives (e.g., linking cubes, base-ten blocks, fingers), but they were not required to use them.

As students responded to each item, the teachers attended closely to the students' behaviors and explanations. They took detailed notes about the students' approaches and explanations and considered whether their thinking processes fit the description of each of the following named strategies: *direct modeling*, *counting*, *fact recall—derived facts*, and *fact recall—known facts*[2]. The

1 Note. The teachers did not ask students to explain why they solved the problem a certain way. Rather, they asked the less judgmental question of how they solved it (or what happened in their minds as they were solving the problem).

2 The descriptions of strategies presented here are the current descriptions used by our team, and we consider

5 + 5	7 + 7
7 + 3	7 + 5
8 + 3	7 + 6
2 + 7	9 + 7
6 + 4	5 + 7
5 + 6	6 + 8
4 + 8	8 + 9
2 + 8	6 + 9
6 + 6	8 + 7

Figure 1. Items used in the fact-fluency assessment

strategies are explained in more detail in the following section. Because addition is commutative, a single term, *addend*, will be used to describe both numbers used in the addition operation.

Types of Strategies Students Commonly Use to Perform Addition on Single-digit Numbers

A student using a *direct modeling* strategy to solve the problem represents each and every quantity in the problem with some sort of object (e.g., manipulatives, fingers, drawings). For example, when evaluating 8 + 3, a student who uses a *direct modeling* strategy may create a set of eight objects, create a second set of three objects, and then count all of the objects in both sets to determine the sum of 11.

A student using a *counting* strategy uses an approach that does not require representing each and every quantity in the problem concretely but does so by counting by ones or by skip counting (rather than recalling other known facts). For example, when evaluating 8 + 3, the student who uses a *counting* strategy may say the word "eight" and then count forward three counting numbers, "9, 10, 11." Such students may use their fingers when counting on from eight, but the important feature is that they do not represent both of the quantities concretely as they would in a *direct modeling* strategy. In this example, the student represented the set of three things concretely, but the eight was held in some sort of mental representation.

A student using *fact recall—derived facts*[3] uses a related, known fact to help solve a problem involving an unknown fact. For example, when evaluating 8 + 3, the student may know the fact 8 + 2 = 10. The student would then need to add only one more to find the final sum of 11.

A *fact recall—known fact* strategy involves the student's recalling the relevant fact directly from memory. When asked how they got the answer, students often respond that they "just knew" the answer.

Assessment of the Students in This Classroom

After the students completed the set of problems, the teachers classified the student according to the categories listed above. The teachers observed that very few students applied the same strategy on every problem, but they reported the strategy each child used most often and wrote the student's name in the corresponding column. Figure 2 shows a representation of their classification of the students in the class.

Looking at the class overall, the teachers noticed that the vast majority of students used a *counting* strategy or a *direct modeling* strategy. Teachers were surprised at the lack of *fact recall—derived fact* and *fact recall—known fact* strategies. Some students did use them, but their use was occasional and inconsistent.

Although *fact recall—derived facts* and *fact recall—known facts* were not the most prevalent strategies used by Eliza, Issabella, Lulu, and Justin, the teachers noted that these were the only students in the class who used strategies based on recalled facts. They wrote each of these students' names in two different columns, because they intended to use the these students' strategies as a starting point for the discussion around these types of strategies during the subsequent

them fluid, as our understanding of these ideas continues to evolve. For a more detailed discussion of these terms, consider reading Carpenter et al. (2015).

3 Students may use many other strategies that involve *derived facts* while solving number-fact problems. For more information about *derived facts*, read chapter three of Carpenter et al. (2015).

Direct modeling	Counting	Fact recall—derived facts	Fact recall—known facts
Emma	Ameree	*Eliza	*Lulu
Logan	*Issabella	*Issabella	*Justin
*Eliza	Landen		
*Justin	*Lulu		
Travis (problems 5–9)	Sarah		
Cesira	Lizzy		
Alyssa	Travis (problems 1–5)		
Lily	Brogan		

Note.
*Four students made use of fact recall in some instances. These four students were placed in two columns as a result of teacher discussions, because these students were identified as being potentially useful for helping other students begin using fact-recall strategies in the subsequent lesson.

Figure 2. Classification of the students on the basis of the strategies they most often used

classroom lesson.

The teachers noticed that Brogan appeared to exhibit some level of understanding of the commutative property of addition. He began his count with the larger number on every problem in which that strategy was applicable. In the interview, he explained, "it's easier to count by a smaller number." Other teachers noted that Lizzie, Landon, Sarah, Travis, Ameree, Lewis, and Eliza also seemed to have an intuitive understanding (or correctly inferred or guessed) that addition is commutative, because they started counting from the larger addend in some their strategies.

The teachers reported some interesting conversation with students about having "always" to start with the big number. One student (Eliza) said, "you don't always have to start with the big number—you can start with the smaller number and still get the same answer." She used 8 + 2 as an example. The teachers discussed how this yields insight into students' informal understanding of the commutative property of addition.

The teachers also noticed that the students who used strategies other than *direct modeling* or *counting* usually used a doubles fact or a sums-of-tens fact they knew to determine the final solution. These two variations of a *fact recall—derived fact* strategy were the most commonly encountered. The teachers decided that if children don't know doubles facts or sums-of-ten facts at recall level, they would be very unlikely to use a *fact recall—derived fact* strategy, because this strategy most often relies on those two categories of facts.

Setting a Learning Goal for These Students on This Day

The teachers discussed the following three approaches they might use to help students develop their ability to use more abstract, fact-based strategies.

1. Provide more opportunities for students to work with sums of ten, because the students seem to rely heavily on *direct modeling* strategies and some of the doubles facts. The experiences should be helpful in getting the students to know the combinations of two numbers that sum to ten by recall.

2. Provide more opportunities for students to work with doubles facts and near-doubles facts as a vehicle to encourage them to begin using *fact recall—derived fact* strategies.

3. Provide opportunities for students to decompose numbers in a variety of ways. The teachers thought these experiences would also help encourage more students to use *fact recall—derived fact* strategies.

The group ultimately wanted to set the learning goal that would best benefit the entire class. The teachers decided that they could work on all three approaches (listed above) in one lesson.

The team agreed on the following learning goal.

> *Students will use facts they can recall (e.g., doubles facts, sums of ten) to generate* ***fact recall—derived fact*** *strategies with understanding based on the composing and decomposing of addends.*

The teachers thought this goal was the next natural step for those students who relied on a *counting* strategy as well as the students who mostly used *fact recall—derived fact* strategies. The discussion moved to the students who used a *direct modeling* strategy. The teachers discussed each individual student and decided that this learning goal could provide opportunities for them to advance toward the learning goal if the problems were chosen or written carefully and deliberately. The teachers agreed that Logan might be the only student for whom this goal might be out of reach.

Planning for the Lesson

...if you can make sense of someone else's strategy (even if you aren't fluent with it yet), you are beginning to think about how to use it in the future.

Guided by this learning goal, the teachers started planning a lesson to implement with these students on the same day as the interviews. They decided to build the lesson around pairs of related expressions, the first pair of which appear below. They thought this initial sequence of problems would be accessible to all students, and they could highlight the relationship between the two problems.

3 + 3
3 + 4

The teachers discussed how students would be likely to solve 3 + 4 and they made notes to watch for students who used 3 + 3 as a starting point for solving 3 + 4. They also assumed that others might count on from three or count on from four. With this sequence, the teachers were hoping to find students who used a *direct modeling* strategy relating 3 + 3 to 3 + 4 as well as students using a *fact recall—derived fact* strategy where they relate the known fact, 3 + 3 = 6, and note that 3 + 4 = 7, because it is one more.

The team then moved to the next step of writing a series of expressions that would help students use the *fact recall—derived fact* strategy of making a ten. The teachers initially considered relying on the 3 in the previous set of expressions and suggested 3 + 7 and 3 + 8. The teachers decided that 5 + 5 was a better transition from the previous set and discussed the benefits of this problem. The discussion next centered on whether 5 + 6, 5 + 7, or 6 + 5 would make the relationships among the problems easiest for students to see. Some teachers thought that 5 + 6 might be too much like the previous set. Others thought 5 + 6 would be a useful problem and that the students might see connections between that expression and the expressions used in the first set. Many teachers thought that 6 + 5 would require students to lean on the commutative property and might not lend itself to connecting to 5 + 5. The team decided to encourage the teacher to make an in-the-moment decision whether to use 5 + 6 or 5 + 7, depending on what the students did on the first set of problems.

The discussion moved to the next pair of expressions and included 6 + 4 with the possibility of seeing that it is equivalent to 5 + 5. The teachers agreed that this equivalence might be difficult for these students to understand. Ultimately, they decided to pose a new set involving 9 + 1 and 9 + 3. They felt this pair was accessible for the range of learners and was not exactly like the previous pairs of problems.

For the last problem, the group of teachers decided that 8 + 4 might be useful, because many students decompose the smaller amount into equal or near equal amounts. In this case, splitting the four into two and two, would be advantageous because it would make a ten.

Lesson Plan

This lesson was developed on the basis of the following learning goal.

> *Students will use facts they can recall (e.g., doubles facts, sums of ten) to generate **fact recall—derived fact** strategies with understanding based on the composing and decomposing of addends.*

3 + 3
3 + 4

5 + 5
5 + 7

9 + 1
9 + 3

8 + 4

Figure 3. The sequence of items teachers decided to use during the planned lesson with the goal of encouraging students to begin using fact-based strategies to evaluate addition expressions

The lesson was designed to consist of a series of events, which would be repeated a number of times. The teacher would provide the expression, allow time for the students to work on the problem, and then orchestrate a class discussion about the strategies the students reported using. Each cycle was designed to take approximately six to nine minutes and repeat three times (once for each pair of problems with the final problem on its own). The sequence of expressions to be posed for students to evaluate is provided in Figure 3.

Initial Pair of Related Expressions

1. Show the first expression (3 + 3) to the class by writing it on chart paper. Instruct the students to solve the problem in whatever way makes sense to them.

2. After the students solve the problem, invite a few students—one at a time—to share their solution strategies with their peers. Record those strategies in a display for students to see.

3. Put the next expression (3 + 4) on the chart paper. Instruct the students to solve the problem in whatever way makes sense to them.

4. After the students solve the problem, invite a few students to share their solution strategies. Focus attention on students who are making connections between 3 + 3 and 3 + 4 by using a *fact recall—derived fact* strategy, which might sound like, "I knew that 3 + 3 was six, so 3 + 4 had to be seven, because it's just one more than six."

Next Pair of Related Expressions

5. Show the third expression (5 + 5) to the class by writing it on chart paper. Instruct the students to solve the problem in whatever way makes sense to them.

6. After the students solve the problem, invite a

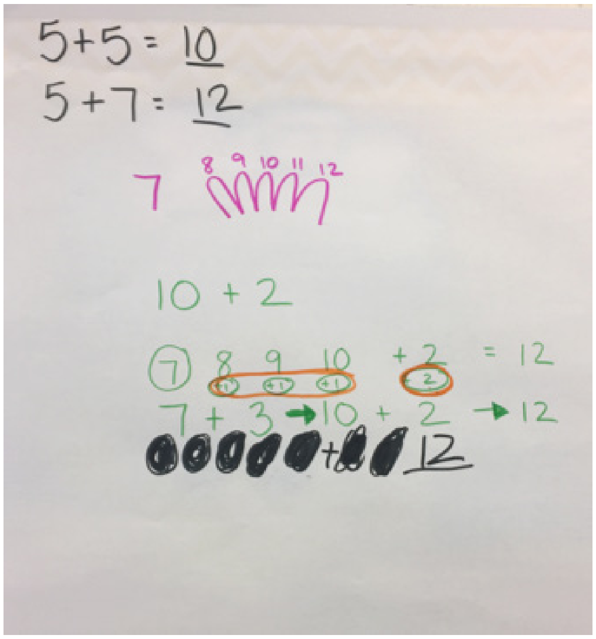

Figure 4. Landen and Sarah's strategy for solving 5 + 7 is shown in green

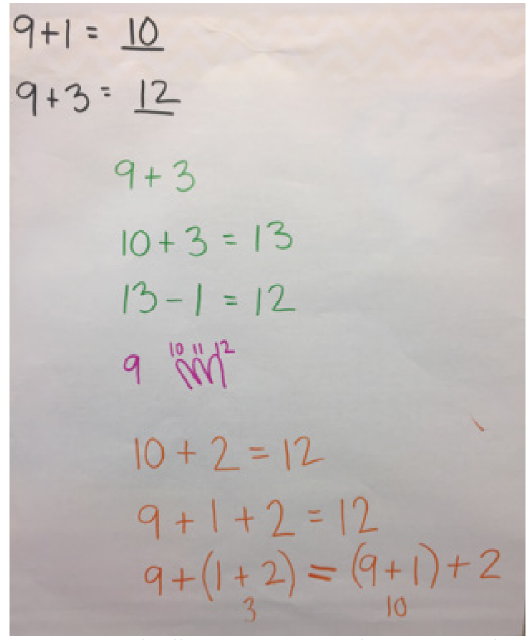

Figure 5. Issabella's strategy is shown in red with orange parenthesis

few students to share their solution strategies.

7. Put the next expression (5 + 7) on the chart paper. Instruct the students to solve the problem in whatever way makes sense to them.

8. After the students solve the problem, invite a few students to share their solution strategies. Focus attention on students who are making connections between 5 + 5 and 5 + 7 by using a *fact recall—derived fact* strategy, which might sound like, "I knew 5 + 5 was 10, so 5 + 7 had to be 12, because this time it's two more."

Third Pair of Related Expressions

9. Write the fifth expression (9 + 1) on the chart paper. Instruct the students to solve the problem in whatever way makes sense to them.

10. After the students solve the problem, invite a few students to share their solution strategies.

11. Put the next expression (9 + 3) on the chart paper. Instruct the students to solve the problem in whatever way makes sense to them.

12. After the students solve the problem, invite a few students to share their solution strategies. Focus attention on students who are making connections between 9 + 1 and 9 + 3 by using a *fact recall—derived fact* strategy, which might sound like, "I knew that 9 + 1 was ten, so 9 + 3 had to be 12, because there is two more still to add."

Final Expression to Evaluate

13. Finally, ask the students to evaluate 8 + 4 and encourage them to solve the problem in whatever way makes sense to them. Consider inviting more students to share their thinking on this problem if students used a variety of strategies Be sure to spend some time focusing on strategies that use a *fact recall—derived fact* strategy, which might take any of several forms. The following list does not cover all possible strategies, but watch for these:

    a. 8 + 2 = 10, and 10 + 2 = 12
    b. 8 + 3 = 11, and 11 + 1 = 12
    c. 8 + 1 = 9, 9 + 1 = 10, 10 + 2 = 12
    d. 8 + 5 = 13, so it's one less, 12
    e. 6 + 4 = 10, and 10 + 2 = 12

Reflection

The group of teachers held a discussion for about one hour to share their observations of the lesson that was enacted. They agreed that the sequence of expression successfully highlighted the usefulness of using doubles and sums of tens to evaluate related expressions. The lesson itself worked well, and the involvement of students communicating their thinking with their peers was an important component. They conjectured that small-group or one-on-one follow up with students may further enhance the learning experience for some students in future lessons.

During the lesson, the lead teacher asked a few times whether a student could repeat how another student solved the problem. This was a new teaching strategy for many teachers in the group, and they thought this was a powerful approach to encouraging students to make sense of various solution strategies. The teachers' discussion led to the following conclusion; if you can make sense of someone else's strategy (even if you aren't fluent with it yet), you are beginning to think about how to use it in the future.

The teachers noticed that two students, Landen and Sarah, made some major changes to how they were solving the problems. Both Landen and Sarah moved from using mostly *counting* strategies to sharing known fact—derived fact strategies when they shared. They both did this really well with the 5 + 5 and 5 + 7 combination. The team noted that other students might also have made some of these connections, but knowing for sure was difficult on the basis of a whole-group discussion. Figure 4 shows a representation of how students like Landen and Sarah were solving 5 + 7 using the known fact of 5 + 5.

Issabella was able to use an interesting *fact recall—known fact* strategy to evaluate 9 + 3. She noticed that 9 + 3 was close to 10 + 3, and she knew the fact that 10 + 3 = 13. So, Issabella took 10 + 3 = 13 and subtracted one from the sum to determine that 9 + 3 = 12. See Figure 5 for a representation of her thinking.

The teachers were somewhat disappointed in the variety of strategies that students used for 8 + 4; most relied on 8 + 2 = 10 and 10 + 2 = 12, and the teachers felt that one more follow-up problem here would have allowed for additional connections. The group agreed that the discussion could have been enhanced if they had considered using 8 + 5 as a follow-up problem to 8 + 4, but they acknowledged that this was only one lesson, and students will need some time to embrace and understand these ideas fully.

What's next?

In discussing the possible next steps for this group of students, the teachers all agreed that they would benefit from more experiences evaluating expressions of the form 9 + ___. These experiences would open the door for them to begin using various making-tens strategies with understanding. The team discussed using Join—Result Unknown word problems involving nine as the initial quantity so that the structure of the word problem would support learning with understanding (rather than introduce additional complexity). An example follows:

> *Adaline had nine stickers. Her mom gave her four more stickers. How many stickers does she have now?*

The team discussed the importance of knowing about individual students and the strategies they use to solve mathematics problems. This understanding can help the teacher make well-reasoned decisions about whom to ask to share in group discussion and could also help with celebrating the variety of strategies in the classroom.

What's Next?

Stories of teachers engaging in collaborative inquiry focused on using student thinking to inform instructional decisions

What's Next? is a collection of stories documenting professional development experiences shared by elementary teachers working collaboratively to study the complex process of teaching and learning mathematics. Each story in the collection describes practicing teachers studying the thinking processes of real students and using what they learn about those students to make decisions and try to help advance those students' understanding on that day.

The teachers in each story start by learning about how individual students are solving a set of mathematics problems. They use this freshly gathered knowledge of student thinking to develop near-term learning goals for students and a lesson plan tailored to specific students on that specific day. One of the teachers implements the planned lesson while the other teachers observe in real time. The teachers then gather to discuss and reflect on their observations and insights.

In these lessons, the practice of teaching is slowed way down. The stories tell of teachers who are studying student thinking and using that information to plan and implement instructional decisions at a pace that is much slower than it occurs in daily practice. The stories in this collection also depict many aspects in common with formative assessment and lesson study, both of which are a process and not an outcome.

The stories depict real situations that occurred in real time and include both successes and shortcomings. We hope that the stories may be studied and discussed by interested educators so that the lessons and ideas experiences of these teachers and instructional coaches may contribute to additional learning and sharing among other interested teachers.

Learn more about these and other stories at **http://www.teachingisproblemsolving.org/**