Moving on From *Counting On*: Developing Addition Fact Fluency

This story is a part of the series:

What's Next? **Stories of Teachers Engaging in Collaborative Inquiry Focused on Using Student Thinking to Inform Instructional Decisions**

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Stories of teachers engaging in collaborative inquiry focused on using student thinking to inform instructional decisions

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Introduction

In the context of a professional-development experience, a group of teachers designed a lesson for a second-grade class that was intended to help students become more fluent with addition facts. The teachers first interviewed the students to learn how they were solving addition-fact problems. On the basis of their observation that the most prevalent strategy being used by students was *counting on*, a lesson was designed to encourage students to use the facts they know to derive unknown facts.

Relevant Florida Mathematics Standards

MAFS.2.OA.2.2 Fluently add and subtract within 20 using mental strategies. By the end of grade 2, know from memory all sums of two one-digit numbers.

Background Information

Consider reading chapter three in *Children's Mathematics: Cognitively Guided Instruction* (Carpenter et al., 2015). This chapter provides background on the varied ways children solve addition and subtraction problems, including the use of known- and derived-fact strategies. It also provides further explanation of the strategies explained in the Analyzing Student Thinking section of the present document.

An alternate source for information on fact fluency is the article "Fluency with basic addition" (Kling, 2011), which discusses components of fluency and promotes the development of fact-based strategies that leverage knowledge of relationships among facts. For more information on assessing fact fluency, consider reading "Assessing basic fact fluency" (Kling & Bay-Williams, 2014), which provides a variety of ways to assess students' fluency with basic math facts.

Carpenter, T. P, Fennema, E., Franke, M. L., Levi, L., & Empson, S. B. (2015) *Children's Mathematics: Cognitively Guided Instruction*. Portsmouth, NH. Heinemann.

Kling, G. (2011). Fluency with basic addition. *Teaching Children Mathematics, 18*(2), 80–88.

Kling, G., & Bay-Williams, J. M. (2014). Assessing basic fact fluency. *Teaching Children Mathematics, 20*(8), 488–497.

Analyzing Student Thinking

In the context of a professional development experience, a group of teachers conducted short one-on-one interviews with all of the students in a second-grade class near the beginning of the school year. The purpose of the interview was to gain insight into the second graders' fluency with addition facts. The interview involved verbally posing a series of addition problems one at a time. If the student's strategy was not readily apparent, the interviewer asked the student to explain how he or she arrived at the answer to each problem. The interviewer had the flexibility to skip any problem(s) judged to be too challenging for the particular student. During the interview, the students did not have access to manipulatives (linking cubes, base-ten blocks, fingers).

Fact Fluency Interview Items

Students were asked to evaluate each one of the following items in Figure 1. The items were presented in order from the top to be bottom of the left-hand column and then from the top to the bottom of the right-hand column.

5 + 5	7 + 7
7 + 3	7 + 5
8 + 3	7 + 6
2 + 7	9 + 7
6 + 4	5 + 7
5 + 6	6 + 8
4 + 8	8 + 9
2 + 8	6 + 9
6 + 6	8 + 7

Figure 1. Set of items used in the fact-fluency interview.

As students responded to each item, the interviewer made note of the details of the strategy used (e.g., for 8 + 3, the student said "9, 10, 11" and put up one finger for each counted number). After the interview, the teachers reflected on the students' strategies using various named categories: *direct modeling, counting on from first, counting on from larger, fact recall—derived facts*, and *fact recall—known facts.*[1]

Named Strategies Commonly Used to Solve Single-digit Addition Problems

A student using a *direct modeling* strategy to solve the problem represents each quantity in the problem with a set of some sort of object (e.g., manipulatives, fingers, drawings). For example, when solving 6 + 8, the student who directly models will create a set of six objects and a set of eight objects and then will determine the answer by counting all starting at one or counting on from one of the sets.

A student using a *counting on from first* strategy determines the sum by counting on from the first number presented in the problem. For the problem 6 + 8, the student would hold the quantity six mentally and count on eight counts (from seven to 14), usually keeping track of the counts with fingers, tally marks, or objects.

A student using a *counting on from larger* strategy determines the sum by counting on from the larger number presented in the problem. For the problem 6 + 8, the student would hold the quantity eight mentally and count on six counts (from nine to 14), again usually keeping track of the counts with fingers, tally marks, or objects. This strategy suggests the student has some understanding of the commutative property of addition ($a + b = b + a$) and uses that knowledge to solve the problem more efficiently.

A student using *fact recall—derived facts*[2] uses a related known fact to help solve a problem involving an unknown fact. For example, when solving 6 + 8, the student might decompose the eight into a four and four, add the first four to six to get 10, and then add the remaining four to the 10 to get the final solution of 14. In this case, the student is using the known facts 6 + 4 = 10, 8 = 4 + 4, and 10 + 4 = 14 to derive the unknown fact 6 + 8 = 14.

A *fact recall—known fact* strategy involves the student's recalling the relevant fact directly from memory. Answers are typically provided quickly. When asked how they got the answer, students often respond that they "just knew" the answer.

Examining the Strategies Used by Students in This Class

After the teachers analyzed students' strategies for each addition-fact problem in relation to the aforementioned categories, they created a chart summarizing their findings (see Figure 2). They recorded the most prevalent strategy used by every student, recognizing that most students used more than one strategy type in the interview.

The teachers noticed that the majority of students used *counting* strategies most prevalently—either *counting on from first* or *counting on from larger*, but they also noted that these students did use *fact* strategies some of the time. Several of the students whose most prevalent strategy was counting used fact-recall strategies for problems involving "doubles" facts (e.g., 5 + 5, 6 + 6). A few of the students classified under *counting* strategies used fact knowledge to approach "make ten" facts (e.g., 8 + 2, 6 + 4). Conversely, students classified as using *fact recall—derived facts* as their most prevalent strategy were sometimes noted to use *counting on from larger* to determine the sum of other facts (e.g., 7 + 6, 6 + 8, 8 + 7).

In reflecting on these data, the teachers observed that the students in the class knew quite a few facts (particularly the doubles facts) but were not using them to derive unknown facts. The teachers

1 The descriptions of strategies presented here are the current descriptions used by our team, and we consider them fluid, as our understanding of these ideas continues to evolve. For a more detailed discussion of these terms, consider reading Carpenter et al. (2015).

2 Students may use many other strategies that involve derived facts while solving number-fact problems. For more information about derived facts, read chapter three of Carpenter et al. (2015).

Direct modeling	Counting on from first	Counting on from larger	Fact recall–derived fact	Fact recall–known facts
Nikki	Leah	Clarice	Cayden	
	Evan	Stephen	Jocelyn	
	Brandon	Nikki	Carter	
	Leticia	Morgan	Nohel	
	Marquis	Gabriel	Tristan	
		Elisa		
		Sabrina		
		Mikel		
		Tatianna		
		Marissa		
		Shane		

Figure 2. Classification of the students on the basis of the strategies they used most often.

therefore developed the following learning goal:

> *Students will notice relationships among addition facts and use known facts to derive unknown facts.*

Planning for the Lesson

So as to increase students' facility with *fact recall—derived fact* strategies, the teachers set out to design a lesson that would build on the basic fact knowledge that students already had. Many students in this class had used a strategy based on recall of known facts on one or more items involving doubles facts in the interviews. The teachers therefore decided to focus on doubles facts and facts that children often derive from doubles facts. They designed a whole-class lesson in which students would discuss and compare strategies for finding the sum of a sequence of addition facts. The teachers reviewed some interesting data gathered during the interviews and generated the following sequence of problems:

5 + 5
5 + 6
6 + 6
8 + 8
If time allows: 8 + 9 or 6 + 9

The teachers decided to open with 5 + 5, because it was the fact for which the most students used the *fact recall—known fact* strategy in the interview. The teachers conjectured that some students who used a *counting* strategy for 5 + 5 during the interview probably knew that fact and were capable of using it without counting on. The teachers discussed a need to make it explicit to students that replacing *counting* strategies with more efficient *derived fact* strategies is an important goal. At the same time, the teachers acknowledged the importance of valuing and honoring students' strategies at all points along the developmental spectrum. With 5 + 5, the teachers agreed that the lesson should elicit students' ideas about multiple ways to determine the sum and begin to provoke thinking about the relative efficiency of the various strategies proposed.

After 5 + 5, the teachers decided to have the students discuss strategies for 5 + 6 and 6 + 6. These particular problems were selected because of their close relationship to 5 + 5. In both cases, teachers wanted to prompt students to think about how knowing 5 + 5 = 10 could be used to figure out 5 + 6 and 6 + 6. The teachers identified particular students in the class who had used strategies during the interview that they would like to be aired during the discussion. For example, for the problem 5 + 6, the teachers noted that Leah had used *counting on from larger* and Sabrina and Caiden has used two different *fact recall—derived fact* strategies. Sabrina's *derived fact* strategy involved substituting (5 + 1) for the 6 so that 5 + 6 could be thought of as 5 + 5 + 1 (which exemplifies how to leverage 5 + 5 to eval-

The teachers agreed on the importance of exposing the class to multiple ***fact recall—derived fact*** *strategies for a given problem to demonstrate and validate that many correct ways exist to use relations among number and operations that can be used to derive unknown facts.*

uate 5 + 6), but Caiden's *fact recall—derived fact strategy* for 5 + 6 did not leverage 5 + 5. Instead, it involved decomposing the 5 into 4 and 1, so that 5 + 6 could be thought of as 6 + (4 + 1), which can then be evaluated as (6 + 4) + 1. The teachers agreed on the importance of exposing the class to multiple *fact recall—derived fact* strategies for a given problem to demonstrate and validate that many correct ways exist to use relations among number and operations that can be used to derive unknown facts.

Next, the teachers determined that the class would discuss how to use number-fact knowledge to approach 8 + 8. This expression was selected, because it was one for which a few students used *fact* strategies in the interview, although most students used *counting on*. The teachers also reasoned that this expression could be solved through a variety of *fact recall—derived fact* strategies and might stimulate a rich discussion.

Recognizing that the 40 minutes allocated for the lesson would probably be consumed by discussion of strategies for the first four problems, the teachers identified two additional problems to pursue if time allowed (i.e., 8 + 9, 6 + 9). These problems were selected because the teachers thought they were closely related to the other problems of focus in the lesson.

Strategy for differentiation to meet the needs of all students in the class

This lesson was developed to center on a whole-group discussion designed to allow the students time to work on and discuss a series of addition facts. Although the desired instructional focus was on fact recall—derived fact strategies, the lesson was intended to incorporate attention to direct modeling and counting strategies used by students. In doing so, the lesson offers access to students at a range of points along the developmental spectrum.

In keeping with the intention to design and implement a lesson that would benefit all the students, teachers discussed how to ensure that the lesson would advance the thinking of students already using *fact recall—derived fact* strategies. To this end, the teachers planned to have students who solved the problems very quickly think about other ways to solve them. Along with the rest of the class, these students would be encouraged to explain and justify their own strategies as well as strategies used by others. They would also be expected to articulate similarities and differences among the various strategies shared.

Notes on what to notice about student thinking

During the lesson, the teachers primarily wanted to determine whether students who used counting strategies in the interview could explain the *fact recall—derived fact* strategies of others and whether they could generate such strategies themselves. For students already using *fact recall–derived fact* strategies, the teachers wanted to attend to whether the children could relate their strategies to those of others and whether they could demonstrate flexible thinking by generating multiple *derived fact* strategies for a given problem.

Lesson Plan

This lesson was developed on the basis of the goal set in the Analyzing Student Thinking section:

> *Students will notice relationships among addition facts and use known facts to derive unknown facts.*

It was designed to repeat the following four steps: (1) the teacher provides the expression for students to evaluate; (2) the teacher allows time for the students to work on the problem; (3) the teacher observes students solving the problem and plans whom to ask to share their thinking; (3) the teacher directs a class discussion about the strategies the students used to solve the problem, with the goal of providing students with practice at expressing their thinking and making sense of their peers' thinking. Each cycle of four steps should take approximately six to nine minutes and can be repeated up to six times (once for each of the following problems).

5 + 5
5 + 6
6 + 6
8 + 8
If time allows: 8 + 9 or 6 + 9

1. Open with a brief discussion of 5 + 5.
   a. Write 5 + 5 on the board.
   b. Ask: "What do you think about when you see 5 + 5?"
   c. Elicit different ideas from students so as to bring out the following ideas.
      i. How two hands with five fingers on each can help you find the sum (direct modeling);
      ii. How counting on, "five… six, seven, eight, nine, ten," extending a finger for each count, can be used to find the sum;
      iii. How some people "just know" that the sum is 10.

2. Facilitate discussion of the ways in which 5 + 5 and 5 + 6 are related problems.
   a. Write 5 + 6 on the board.
   b. Ask: If you know 5 + 5, could you use that information to solve 5 + 6?
   c. Have students turn to a neighbor and discuss the question above. As students discuss, identify students who are articulating the *fact recall—derived fact* strategy involving thinking of six as 5 + 1 (i.e., 5 + 6 = 5 + (5 + 1) = (5 + 5) + 1 = 11).
   d. Have multiple students explain the above-mentioned *fact recall—derived fact* strategy and alternate *derived fact* strategies. As students describe these strategies, introduce notation for recording the strategy. (See the Reflection section for examples of notation.)
   e. Spend ample time on this problem, having students explain and justify how

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they “broke apart” and “put together” the numbers to make determining the sum easier. Also, have students explain how your notation is related to the way the numbers were “broken apart” and “put together.” Relate the strategy and notation to direct modeling if some are using this approach (e.g., “I am thinking of it as five on one hand, five on the other hand, and then one more finger.”).

3. Facilitate discussion of multiple strategies for approaching additional problems (6 + 6, 8 + 8, 8 + 9, and 6 + 9), focusing on one problem at a time. For each problem:
   a. Write the expression on the board.
   b. Encourage students to try to determine the sum on their own.
   c. After allowing them to think individually and watching individuals as they work to assess their thinking, direct students to talk to a neighbor about different ways they can “break apart” and “put together” the numbers to make finding the sum easier. Ask: How can you use an “easy to remember” fact to help you figure out this fact?
   d. As students discuss with a partner, listen for different *fact recall—derived fact* strategies. Identify students to share their strategies with the whole class.
   e. Through whole-class discussion, have select students share their strategies while you record them on the board. Consider sequencing students’ strategies in order of sophistication (with *direct modeling* or *counting* strategies shared first and *fact recall—derived facts* strategies shared later). Invite other students to explain their classmates’ strategies and interpret the notation used to record the strategies.
   f. Invite students to note similarities and differences in strategies used for a given problem. Guide students to focus on how the numbers are “broken apart” and “put together” differently. This approach is also likely to stimulate an observation that *fact recall—derived fact* strategies are more efficient than *counting on* and *direct modeling*.

One student explained that she liked ***counting on****, because it worked every time. For her, it was a strategy with which she was comfortable and confident.*

4. Close the lesson by asking students to reflect on what they have learned or what they want to remember from today’s lesson. Invite students to share their ideas.

Reflection

What we learned about the students

As the lesson progressed, most of the second-graders were able to explain verbally the *fact recall—derived fact* strategies for evaluating the expressions. Some of the students who were primarily using *counting* strategies in the interview did make a shift during the lesson to begin generating their own *derived fact* strategies for some of the later problems. Others who primarily used the *counting on* strategy in the interview continued to use *counting on* as their own first way to determine the sum. One student explained that she liked *counting on*, because it worked every time. For her, it was a strategy with which she was comfortable and confident. They conjectured that students like this one would, with more opportunities to experience and explain *derived fact* strategies, eventually replace *counting on* strategies with the more efficient *derived fact* strategies as she grew more confident in her knowledge of facts and relations among them.

Reflections on the lesson

During the lesson, the teacher introduced a way of notating students' *fact recall—derived fact* strategies that appeared to help students to make sense of how each strategy worked. Specifically, the teacher used parentheses to highlight how each strategy "broke apart" and "put together" the numbers. To offer illustration, Figure 3 presents the teacher's notation of three different *fact recall—derived fact* strategies students shared for 8 + 8.

Early in the lesson, the teacher simply modeled the notation and asked questions to facilitate the students' making sense of it. In the moment, it seemed to the teacher that parentheses—which are a standard and conventional component of mathematical notation—could be useful in expressing the students' ideas in writing.

She asked the class whether they had ever used parentheses in their math equations. (They had not used them or been exposed to them previously.) She asserted that parentheses can be really useful in making it clear how a strategy works. The teacher posed questions such as, "Why do you think I put these numbers in parentheses?" and "How does this way of writing it show how Carter thought about the problem?" By the time students were working on generating strategies for 8 + 8 toward the end of the lesson, they were suggesting how a given strategy should be expressed in writing, and they adopted the conventionally correct use of parentheses in their own notation almost immediately. This result was very interesting to the teachers, some of whom said they were surprised to see the students making sense of the notation with parentheses so readily.

The teachers also discussed next steps for instruction with this group of students. The teachers noted that, in the discussion of 8 + 8, the students were using strategies involving "making ten." The teachers wondered whether reviewing different ways to make ten (e.g., 2 + 8, 5 + 5, 6 + 4) and exploring different ways to use that knowledge might be helpful. In general, the teachers agreed that the students would benefit from simply having more lessons like this one—lessons that facilitated discussion of strategies and opportunities for students to learn from each other.

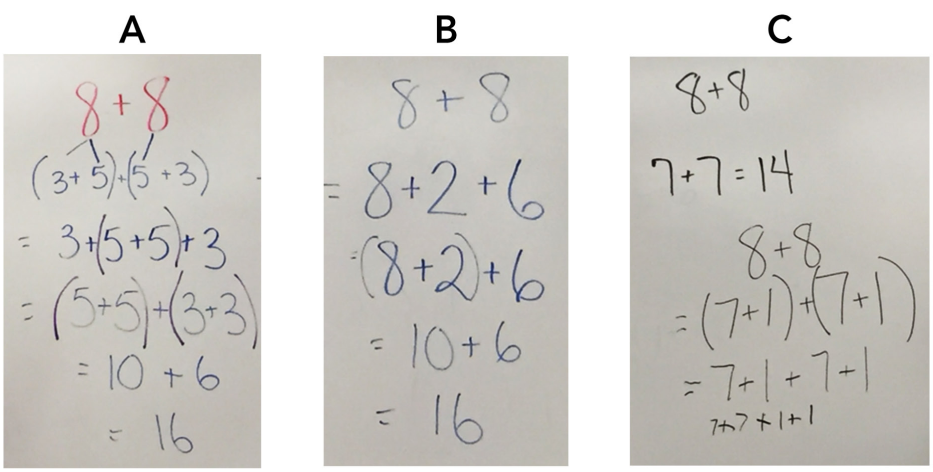

Figure 3. The teacher's notation of three ***fact recall—derived fact*** *strategies for 8 + 8.*

What's Next?

Stories of teachers engaging in collaborative inquiry focused on using student thinking to inform instructional decisions

What's Next? is a collection of stories documenting professional development experiences shared by elementary teachers working collaboratively to study the complex process of teaching and learning mathematics. Each story in the collection describes practicing teachers studying the thinking processes of real students and using what they learn about those students to make decisions and try to help advance those students' understanding on that day.

The teachers in each story start by learning about how individual students are solving a set of mathematics problems. They use this freshly gathered knowledge of student thinking to develop near-term learning goals for students and a lesson plan tailored to specific students on that specific day. One of the teachers implements the planned lesson while the other teachers observe in real time. The teachers then gather to discuss and reflect on their observations and insights.

In these lessons, the practice of teaching is slowed way down. The stories tell of teachers who are studying student thinking and using that information to plan and implement instructional decisions at a pace that is much slower than it occurs in daily practice. The stories in this collection also depict many aspects in common with formative assessment and lesson study, both of which are a process and not an outcome.

The stories depict real situations that occurred in real time and include both successes and shortcomings. We hope that the stories may be studied and discussed by interested educators so that the lessons and ideas experiences of these teachers and instructional coaches may contribute to additional learning and sharing among other interested teachers.

Learn more about these and other stories at **http://www.teachingisproblemsolving.org/**