Cultivating Addition Fact Fluency through Discussion of Strategies

This story is a part of the series:

What's Next? **Stories of Teachers Engaging in Collaborative Inquiry Focused on Using Student Thinking to Inform Instructional Decisions**

What's Next?

Stories of teachers engaging in collaborative inquiry focused on using student thinking to inform instructional decisions

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Introduction

This story describes the development and implementation of a lesson designed to help a class of second-grade students attain greater fluency with addition facts. In the context of a professional-development workshop, a group of teachers interviewed the second graders to learn how they were solving addition-fact problems. Interview findings revealed that students used a wide range of strategies; *counting on* was the most common strategy observed. On this basis, a lesson was designed to provoke the second-grade students to notice number relations and consider how known facts might be used to derive other facts.

Relevant Florida Mathematics Standards

MAFS.2.OA.2.2 Fluently add and subtract within 20 using mental strategies. By the end of Grade 2, know from memory all sums of two one-digit numbers.

Background Information

Consider reading chapter three in *Children's Mathematics: Cognitively Guided Instruction* (Carpenter et al., 2015). This chapter provides background on the varied ways children solve addition and subtraction problems, including the use of *known* and *derived fact* strategies. It also expands upon the strategies discussed here.

An alternate source of information on fact fluency is the article "Fluency with basic addition" (Kling, 2011), which discusses components of fluency and promotes the development of cognitive strategies that leverage knowledge of relations among number facts.

In addition, chapter two in Thinking Mathematically: Integrating Arithmetic and Algebra in the Elementary School (Carpenter, Franke, & Levi, 2003) describes children's conceptions of the equals sign and offers instructional suggestions for helping children to develop an accurate and robust conception.

Carpenter, T. P., Fennema, E., Franke, M. L., Levi, L., & Empson, S. B. (2015). *Children's Mathematics: Cognitively Guided Instruction (2nd. Ed.)*. Portsmouth, NH: Heinemann.

Carpenter, T. P., Franke, M. L., & Levi, L. (2003). *Thinking Mathematically: Integrating Arithmetic and Algebra in Elementary School*. Portsmouth, NH: Heinemann.

Kling, G. (2011). Fluency with basic addition. *Teaching Children Mathematics, 18*(2), 80–88.

Analyzing Student Thinking

In the context of a professional-development workshop, a group of teachers conducted short, one-on-one interviews with each student in one second-grade class as a way to gain insight into the students' fluency with addition facts. The interviews involved verbally posing a series of addition problems one at a time (see Figure 1). If the student's strategy was not readily apparent, the interviewer asked the student to explain how he or she arrived at the answer. The teacher interviewer had the flexibility to skip any problem(s) judged too challenging for the particular student or to shift to an alternative, entry-level list of addition facts, in which only sums within 10 were included. During the interview, students did not have access to manipulatives, but they were allowed to use their fingers if they wished.

5 + 5	7 + 7
7 + 3	7 + 5
8 + 3	7 + 6
2 + 7	9 + 7
6 + 4	5 + 7
5 + 6	6 + 8
4 + 8	8 + 9
2 + 8	6 + 9
6 + 6	8 + 7

Figure 1. Set of addition expressions used in one-on-one interviews (posed in sequence down the left column and then down the right).

Direct modeling	Counting on (from first or larger)	Fact recall–derived facts	Fact recall–known fact
Jeremy	Widalyn	Jayden	–
Rafael	Lumary	Sebastian	
Jeamonte	Paige	Samuel	
Xavier*	Raheem		
Isabella*	Samuel		
	Mily		
	Clayton		

*These two students produced incorrect answers to many items on the addition-fact interview protocol presented in Figure 1, so interviewers elected to pose easier, entry-level addition problems involving sums within 10.

Figure 2. Classification of the students by the most prevalent strategies observed in the addition fact interview.

As students responded to each item, the teacher interviewer made note of the details of the strategy used (e.g., for 8 + 3, the student said "9, 10, 11," while counting on fingers). After the interviews, the teachers reflected on the students' strategies. The teachers' discussion was organized by various named categories of student strategies, including *direct modeling*, *counting on*, *fact recall—derived facts*, and *fact recall—known facts.*[1]

Named Strategies Commonly Used to Solve Single-digit Addition Problems

A student using a *direct modeling* strategy to solve the problem represents each and every quantity in the problem with some sort of object (e.g., manipulatives, fingers, drawings). For example, when evaluating 3 + 5, students who use a *direct modeling* strategy may extend three fingers and five fingers. Then, they will determine the answer by counting all of the extended fingers. When addition sums exceed ten, students find inventive ways to direct model with fingers, such as double-counting certain fingers, counting their finger pads, or visualizing imaginary fingers[2].

A student using a *counting on* strategy determines the sum by counting forward from one of the numbers presented in the problem. For the problem 4 + 7, the student might hold the first quantity (four) mentally and count forward seven counts (from five to 11), usually keeping track of the counts with fingers, tally marks, or objects. Alternatively, the student might choose to hold the larger quantity, seven, mentally and count forward four counts (from eight to 11). This *counting on from larger* strategy suggests the student has some understanding of the commutative property of addition (a + b = b + a), and uses that knowledge to solve the problem more efficiently.

A student using *fact recall—derived facts* strategy uses a related known fact to help solve a problem involving an unknown fact. For example, when solving 6 + 8, the student might decompose the eight into four and four, add the first four to six to get ten, and then add the remaining four to the ten to get the final solution of 14. In this case, the student is using the known facts 6 + 4 = 10, 8 = 4 + 4, and 10 + 4 = 14 to derive the unknown fact 6 + 8 = 14.

A *fact recall—known fact* strategy involves the student's recalling the relevant fact directly from memory. Answers are typically provided quickly. When asked how they got the answer, students often respond that they "just knew" the answer.

After the teachers analyzed students' strategies for each addition problem, they identified the most prevalent strategy used by each student (recognizing that most students used more than one strategy type in the interview). They created a chart summarizing their findings (Figure 2).

1 The descriptions of strategies presented here are the current descriptions used by our team, and we consider them fluid, as our understanding of these ideas continues to evolve. For a more detailed discussion of these terms, consider reading Carpenter et al. (2015).
2 For more discussion of this latter idea, read Lily's "Invisible Fingers" Strategy for Solving Subtraction Fact Problems.

Variation in Students' Strategies

Through lengthy discussion of how individual students approached different addition facts, several conclusions were reached made about the second-grade class. One-third of the students in the class had used *direct modeling* as their primary strategy. Two of those students did not consistently count accurately. At the same time, all of the students classified as primarily using *direct modeling* reported "just knowing" (using a fact recall-known fact strategy for) one or more facts (e.g., 5 + 5).

The group of seven students who primarily used the *counting on* strategy also knew 5 + 5 at a fact-recall level as well as some combination-of-ten facts (e.g., 7 + 3, 6 + 4). A few students reported "just knowing" the doubles fact 6 + 6, and one student who used a *counting on* strategy was noted to have used *fact recall—derived fact* strategies to evaluate a couple of expressions. All but two of the students who used a *counting on* strategy consistently counted on from larger rather than always *counting on* from the first number presented in a given problem and therefore understood the commutative property of addition on some level.

Of the three students who primarily used *derived fact* strategies, all demonstrated facility with *derived fact* strategies that built on combination-of-ten facts. For example, Samuel approached 8 + 3 by thinking about its relationship to 8 + 2, a combinations-of-ten fact he "just knew." Samuel explained that he solved 8 + 3 by adding one to the sum of eight and two. One student, Jayden, used *derived fact* strategies that flexibly leveraged knowledge of combination-of-ten facts and doubles facts.

Through reflection on what they learned about these students in the interviews, the teachers observed that all of the students in the class knew the 5 + 5 = 10 fact at a recall level, whereas many of the students who were primarily using *counting on* and *direct modeling* strategies knew multiple addition facts at a *fact recall—known fact* level. Most of these second graders were not, however,

4 + 4

5 + 5

6 + 4

6 + 5

7 + 3

7 + 4

9 + 5

Figure 3. Sequence of addition expressions for students to evaluate during the classroom lesson.

using *derived facts* to solve problems. The teachers therefore decided to develop a lesson for the class that focused on working toward the following learning goal:

> Students will consider relations among addition facts and use derived fact strategies to evaluate addition expressions.

Planning for the Lesson

So as to increase students' facility with *fact recall—derived fact* strategies, the teachers decided to design a lesson that would build on the base of known-fact knowledge that many students in the class already had. Because many students knew some combination-of-ten facts at a recall level, they decided to focus the lesson on stimulating exploration of how combination-of-ten facts might be used to derive related addition facts with sums greater than ten. The teachers therefore planned a whole-class lesson in which students would discuss and compare strategies for evaluating a carefully designed sequence of addition expressions. The teachers created the following list of expressions for the students to evaluate.

The teachers selected 4 + 4 as the first fact for discussion, because it would provide a comfortable entry point for students who needed to use *direct modeling* with their fingers. It was also reasoned that this first problem could be used to establish the fact-by-fact discussion routine and behavioral expectations. Specifically, students would be expected to think about their answer and strategy to each problem without "shouting it out." Then, the class would share and compare students' different ways of approaching each problem with a shared intent to understand and learn from one another's thinking. The teacher would keep a record of each strategy on the board, and she would ask the class many questions about the various strategies. The purpose of the teachers' questions would be, simultaneously, to bring students' thinking into the shared space of the classroom, to help children learn to express their ideas in verbal and written representations, and to slow down the pace so that other students in the classroom could mentally process the ideas being shared. The teachers wanted to ensure that a range of strategies were shared for this entry problem, so they identified particular students to call on who would be likely to share *direct modeling*, *counting on*, and *known fact* strategies. This method was intended to accomplish the three simultaneous goals while also providing opportunities to examine connections among the different strategies.

The purpose of the teachers' questions would be, simultaneously, to bring students' thinking into the shared space of the classroom, to help children learn to express their ideas in verbal and written representations, and to slow down the pace so that other students in the classroom could mentally process the ideas being shared.

The next two expressions 5 + 5 and 6 + 4 were selected to provoke discussion of different combinations of ten and to set the stage for encouraging derived fact strategies for the expressions presented later in the sequence. From the interview data, teachers anticipated that all students would know the answer to 5 + 5 at a recall level, so discussion would focus on strategies for evaluating 6 + 4 as well as the equivalence of 6 + 4 and 5 + 5. This point was also considered to be a good one at which to introduce equation notation with parentheses to examine the equivalence of these expressions. The teacher might guide students to discuss the relationships between these facts by examining the decomposition and recomposition of numbers in adjacent expressions of an equation, as in: $6 + 4 = (5 + 1) + 4 = 5 + (1 + 4) = 5 + 5$.

Next, 6 + 5 was selected to provoke discussion of the lesson target, derived fact strategies involving combination-of-ten facts. In particular, the teachers expected that students would identify ways to use 5 + 5 and 6 + 4 to derive the sum of 6 + 5.

The final three expressions selected for inclusion in discussion were 7 + 3, 7 + 4, and 9 + 5. The expressions 7 + 3 and 7 + 4 were selected as a pair. The teachers thought this ordered pair would create a situation where the combination-of-ten fact 7 + 3 could be used to derive the sum of 7 + 4. Then, 9 + 5 was included to offer an opportunity for the teacher to determine whether students would use a *derived fact* strategy without as much immediate scaffolding.

Strategy for differentiation to meet the needs of all students in the class

In general, the teachers discussed the importance of including and honoring a variety of student strategies in the discussion. Although the sequence of facts was designed to foster student use of *derived fact* strategies using combination-of-ten facts, the broader aim of a lesson such as this one is to encourage a reasoning-centered approach to mathematics. Therefore, students using strategies at all points along the strategy continuum should be able to see ideas similar to theirs represented in the discussion. The discussion was conjectured to help students build a bridge between their current strategies and more sophisticated strategies by comparing and contrasting different strategies (or just working to understand them).

During the lesson, the teachers primarily wanted to know (a) whether students who used counting strategies in the interview could understand and explain the fact recall—*derived fact* strategies presented by their peers and (b) whether they could generate such strategies themselves. The teachers wanted students who were already using *fact recall—derived fact* strategies to relate their strategies to the strategies used by their peers and to demonstrate multiple derived fact strategies for a given problem. Forming these connections should yield greater understanding than only considering one way to solve a problem. The teachers also wanted to help students who primarily used *direct modeling* in the interview to understand and explain the more sophisticated *counting on* and *derived fact* strategies used by their peers.

In planning this lesson, the teachers were mindful that the one-third of students who had primarily used *direct modeling* strategies to solve the addition problems in the interview knew only a few of the facts at a recall level. The teachers discussed how, in addition to understanding strategies derived from relations among facts (i.e., *derived fact* strategies), these particular students would benefit from understanding how their classmates were approaching facts using *counting on*. When *counting on* strategies surfaced in the discussion, the teacher would invite the students who used *direct modeling* strategies in the interview to explain or interpret the *counting on* strategies described by their peers (or, at least, to relate them to the *direct modeling* strategies).

The teachers discussed how pictorial representations of *direct modeling* strategies would be useful for demonstrating how *derived fact* strategies could be justified and represented concretely. The teachers planned to introduce *direct modeling* strategies intentionally during the discussion and then to guide students to relate more sophisticated strategies such as *counting on* to the pictorial representations.

Lesson Plan

This lesson was developed on the basis of the goal set in the Analyzing Student Thinking section:

> *Students will consider relations among addition facts and use* derived fact *strategies to evaluate addition expressions.*

The lesson was designed to follow the three-part cycle of launch, work time, and discussion. The cycle would be repeated several times. The basic cycle involves the teacher's providing an expression, allowing time for the students to think about the problem mentally and signal when they have determined the answer, and then facilitating discussion of students' strategies. The cycle is repeated for each of the following expressions:

4 + 4
5 + 5
6 + 4
6 + 5
7 + 3
7 + 4
9 + 5

1. Provide an overview of the lesson and establish behavior expectations.
   a. Tell students that today they are going to work together to think about different ways to solve addition-fact problems.
   b. Emphasize the importance of listening to peers and thinking about what you can learn from them.
   c. Preview the basic lesson procedure: "I will write a problem on the board and give you time to think about your answer. You should not shout out. When you know the answer, you should put a thumbs-up signal on your belly. That will tell me you are ready. Then, while you wait for everyone to be ready, you should think about how you can explain your way of figuring out the sum."
   d. Practice the silent thumbs-up sign to signal readiness, and reemphasize the importance of not shouting out.
2. Facilitate the think-signal-discuss cycle for 4 + 4.
   a. Write 4 + 4 on the board.
   b. Remind students that they should think about the problem silently and display a thumbs-up sign on their bellies when they know the answer. Wait for all students to display a thumbs-up sign before initiating discussion.
   c. Elicit students' varied strategies for 4 + 4, and make a record on the board of each idea. Call on students who are likely to bring up the following ideas:
      i. How two hands with four fingers on each can be used to find the sum (*direct modeling*)
      ii. How counting, "four…five, six, seven, eight," extending a finger for each count, can be used to find the sum
      iii. How some people "just know" that the sum is 8
   d. Use strategies to promote active sense-making and attention to the ideas of peers:
      i. Prompt students to explain/repeat the strategies described by peers
      ii. Prompt students to explain how/why a strategy works or how a written record (using mathematical notation) matches the students' strategy
      iii. Have students compare two strategies. In particular, have students explain the relationship between a *direct modeling* strategy and a more sophisticated strategy
3. Facilitate the think-signal-discuss cycle for 5 + 5 and 6 + 4 with emphasis on establishing that these are related combination-of-ten facts.
   a. Write 5 + 5 on the board.

b. Direct students to show thumbs-up when they know the answer. If this happens quickly (as anticipated), invite the class to chorally say the answer. Hold a brief discussion emphasizing that there are some facts we "just know" and move on to 6 + 4. (This plants a seed for discussion of *derived fact* strategies based around known facts.)

c. Write 6 + 4 on the board and follow the think-signal procedure.

d. Elicit students' varied strategies for 6 + 4, and make a visual record of each strategy on the board (see examples in Phase 5).

   i. Ensure inclusion of a *direct modeling* strategy to reference during discussion of more sophisticated strategies.

   ii. (As helpful) Prompt students to relate counting or *derived fact* strategies to the recorded *direct modeling* strategy.

   iii. If it doesn't come up naturally, consider asking: Does knowing 5 + 5 help us know 6 + 4? How?

      » Consider introducing equation notation to examine the equality of 5 + 5 and 6 + 4. Two options that might match students' thinking:

      $5 + 5 = (4 + 1) + (6 - 1) = 6 + 4 + 1 - 1 = 6 + 4$

      $6 + 4 = (5 + 1) + 4 = 5 + (1 + 4) = 5 + 5$

e. If students have limited experience with equation notation, consider taking a few minutes to probe and discuss students' conceptions of nonstandard equations and the equals sign.

   i. Write 5 + 5 = 6 + 4 on the board. Have students discuss the following question with a neighbor and then as a class: Is this equation true, or is it false? Why?

   ii. Elicit explanation of multiple viewpoints

The teachers discussed how pictorial representations of direct modeling strategies would be useful for demonstrating how derived fact strategies could be justified and respresented concretely.

without judgment. (Note: A common misconception is that 5 + 5 ≠ 6 + 4, because 5 + 5 ≠ 6. This misconception often reflects a partial understanding of the equals sign and related notation.)

iii. Remind students that the equals sign means "the same as," and reframe the question: "Is the sum of five plus five the same as the sum of six plus four?"

» Consider comparing pictures of 5 + 5 and 6 + 4 to explore this question.

» Consider writing 10 = 10 on the board and having students discuss whether the statement is true or false.

4. Facilitate the think-signal-discuss cycle for 6 + 5, with emphasis on exploring *derived fact* strategies.

   a. Elicit a variety of strategies, make a visual record of each (see the next section for examples), and use questioning to help students understand each others' ideas.

      i. If the subject does not come up naturally, prompt students to consider *derived fact* strategies involving the combination-of-ten facts 5+5 and 6+4. Ask: How might knowing 6 + 4 = 10 (or 5 + 5 = 10) help us figure out 6 + 5?

      ii. As students share *derived fact* strategies, record the flow of students' thinking with arrow notation and then help students to understand the decomposition and recomposition of numbers for the given strategy using equation notation. For example:

         - A student says, "I did six plus four to get ten, and then added one. So it is 11."
         - Notation using arrows in the initial recording of the student's thinking: 6 + 4 → 10 +1 → 11
         - Equation notation: 6 + 5 = 6 + (4 + 1) = (6 + 4) + 1 = 11
         - Related questions:
            1. Student said she did 6 + 4 first. I see the six in 6 + 5…why did she add four?
            2. Why did she add the one to the ten?
            3. So, what number in 6 + 5 got broken apart? How did it get broken apart? Why that particular way?

5. Facilitate the think-signal-discuss cycle for the remaining expressions (7 + 3, 7 + 4, 9 + 5), presenting each one at a time and emphasizing examination of *derived fact* strategies.

   a. Considerations for 7 + 3

      i. Consider limiting the amount of time spent on this one and simply establish that the sum is 10 (as a scaffold for considering a *derived fact* strategy for 7 + 4).

      ii. Consider encouraging students who primarily used *direct modeling* in the interview to attempt using a *counting on* strategy for this one.

      iii. Consider extending earlier discussions of notation related to the equals sign and invite students to discuss whether the following equation is true or false: 5 + 5 = 6 + 4 = 7 + 3

   b. Considerations for 7 + 4

      i. As possible, encourage the class to generate multiple *derived fact* strategies. Ask: Are there any other facts that you know quickly that would be helpful for figuring out this one?

         1. Derived fact strategies most likely to surface:

a. 7 + 4 = 7 + (3 + 1) = (7 + 3) + 1 = 11

b. 7 + 4 = (1 + 6) + 4 = 1 + (6 + 4) = 11

ii. Use arrow notation to record a students' *derived fact* strategy in real time (e.g., 7 + 3 → 10 + 1 → 11). Then, have students suggest how to record the strategies using equations with or without parentheses.

iii. Ask the class questions to clarify how each strategy works.

c. Considerations for 9 + 5. This problem may be more difficult than the previous ones. Here are a few options for its presentation:

i. Pose this problem in the same manner as the previous problems.

ii. Ask individuals or partners to generate two or three ways to solve before sharing with the larger group; eavesdrop on students as they share with their partners so that you can find students to share their strategies publicly.

iii. Have students write about strategies for this expression as an exit ticket.

1. *Derived fact* strategies most likely to surface for this problem:

a. 9 + 5 = 9 + (1 + 4) = (9 + 1) + 4 = 14

b. 9 + 5 = (4 + 5) + 5 = 4 + (5 + 5) = 14

Reflection

Reflections on the lesson

During the lesson, the teacher skillfully used pictorial and symbolic representations of the addition facts to anchor discussion of relations among various strategies and addition facts. In the beginning of the lesson, the teacher invited Isabella, a student who had primarily used *direct modeling* strategies in the interview, to share her way

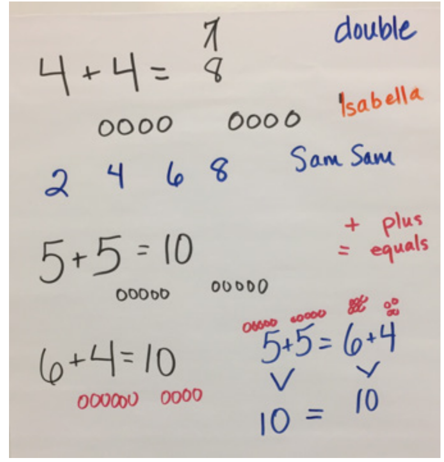

Figure 4. The teachers' record of strategies shared for 4 + 4, 5 + 5, and 6 + 4.

of finding the sum of 4 + 4. Isabella explained how she represented the fours with her fingers and counted them, and the teacher recorded her *direct modeling* strategy by drawing two sets of four circles. (See the teacher's record of students' strategies in Figure 4.) From this point forward in the lesson, this *direct modeling* strategy was referred to as "Isabella's way."

As the lesson progressed, the teacher continually guided the class to "double-check their thinking" with Isabella's way. For example, after some discussion about whether the equation 5 + 5 = 6 + 4 was true or false, the teacher suggested, "let's use Isabella's way to see if the sum of 5 + 5 is the same as the sum of 6 + 4." As the teacher went on to draw the corresponding number of circles for each number in 5 + 5 = 6 + 4 (see Figure 3), she asked, "What do you think…will we have the same number of circles on both sides of the equals sign?" She then had the class chorally count the circles on each side of the equation and emphasized the equality of 5 + 5 and 6 + 4.

This practice of using Isabella's way to verify the equality of expressions was then used again as the class discussed a *derived fact* strategy for 6 + 5. (See the bottom of the teacher's record in Figure 5.) The use of Isabella's *direct modeling* strategy throughout the lesson not only provided a visual support for all students to understand the strategies and equations discussed, it also honored and offered connection to the mathematical thinking of Isabella and the other students whose

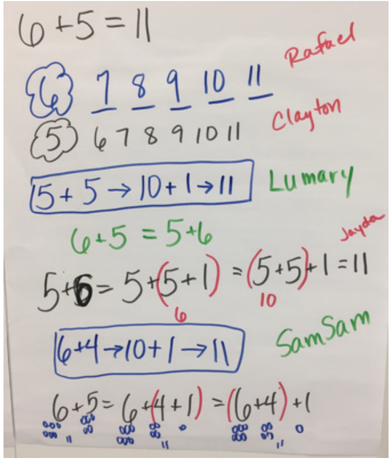

Figure 5. The teachers' record of student strategies for 6 + 5

current approach to solving these problems was to use a *direct modeling* strategy.

The teacher continually used written notation and questions to support the class in making sense of their peers' strategies, particularly *derived fact* strategies. For example, in the discussion of 6 + 5, Lumary shared her strategy, "I started with five, and I added five more to get ten. Then I added one, to get 11." The teacher recorded the flow of Lumary's *derived fact* strategy with deliberate use of an arrow rather than an equals sign (see Figure 4), because some expressions connected by the arrows were not equivalent. She then directed students to talk with a neighbor about why Lumary did 5 + 5 as her first step when the problem was 6 + 5. One student asserted that maybe Lumary was thinking about how 6 + 5 is the same as 5 + 6, so she could start with the five. The teacher recorded this idea on the board and then called on Jayda, who explained further, "it's like five plus six is the same as five plus five plus one." The teacher recorded 5 + 6 = 5 + 5 + 1 on the board, and prompted the rest of the class to consider where the six (from 5 + 6) is in the 5 + 5 + 1 part of the equation. As students identified the part of the expression equivalent to six, the teacher used a red marker to add parentheses (thereby introducing a convention of mathematical notation that students will see in higher grade levels). Then, the teacher added an expression to the equation so it read 5 + 6 = 5 + (5 + 1) = (5 + 5) + 1, and she reminded students that Mily's first step was 5 + 5 = 10. She prompted students to identify that step in the newly added part of the equation, and she again used red marker to add parentheses for the part representing ten (see Figure 4). Through careful attention to representing student ideas with mathematical notation and questions about the notation, the teacher facilitated student understanding of how and why numbers were decomposed and recomposed to solve the target problem.

What we learned about the students

During the lesson, the teachers observed the individual students they interviewed that morning. As the lesson progressed, the teachers observed that most of the individual second graders were able to explain verbally strategies that were more sophisticated than the strategies they used in the interview. For example, several students who primarily used *direct modeling* in the interview were able to explain how a classmate had used *counting on* during the discussion. Students who had primarily used *counting on* in the interview were able to explain and began to generate their own *derived fact* strategies. The teachers also observed that many students continued to favor the strategies they used in the interview, even when they had shown that they were capable of explaining and even generating more sophisticated strategies.

The teachers agreed that this lesson served as a starting place for opening students' eyes to the possibility of more sophisticated strategies. They also agreed that students would need more experiences like this one before those less sophisticated strategies would be replaced by the more sophisticated ones.

What's Next?

Stories of teachers engaging in collaborative inquiry focused on using student thinking to inform instructional decisions

What's Next? is a collection of stories documenting professional development experiences shared by elementary teachers working collaboratively to study the complex process of teaching and learning mathematics. Each story in the collection describes practicing teachers studying the thinking processes of real students and using what they learn about those students to make decisions and try to help advance those students' understanding on that day.

The teachers in each story start by learning about how individual students are solving a set of mathematics problems. They use this freshly gathered knowledge of student thinking to develop near-term learning goals for students and a lesson plan tailored to specific students on that specific day. One of the teachers implements the planned lesson while the other teachers observe in real time. The teachers then gather to discuss and reflect on their observations and insights.

In these lessons, the practice of teaching is slowed way down. The stories tell of teachers who are studying student thinking and using that information to plan and implement instructional decisions at a pace that is much slower than it occurs in daily practice. The stories in this collection also depict many aspects in common with formative assessment and lesson study, both of which are a process and not an outcome.

The stories depict real situations that occurred in real time and include both successes and shortcomings. We hope that the stories may be studied and discussed by interested educators so that the lessons and ideas experiences of these teachers and instructional coaches may contribute to additional learning and sharing among other interested teachers.

Learn more about these and other stories at **http://www.teachingisproblemsolving.org/**