

The Multifaceted Nature of Place Value: It's About More than Digit Values

Robert Schoen: A lot of times when I go into classrooms, I see students writing “h’s” above the digit in the hundreds place and a “t” above the digit in the tens place and a “o” above the digit in the ones place, and it’s pretty clear that they’ve been sort of taught to do that. And that’s certainly one component of a place-value understanding, but I don’t think by itself it’s really the target that we’re trying to get our students to understand about place value and about our number system that’s written in base 10.

Zak Champagne: One main question, which is “what constitutes strong place value understanding?,” particularly in Kindergarten, first-, and second-grade students, we’ll kind of look at 5 understandings that we’ve identified as a team that we think are really important for younger students to engage with and think about.... The team here at Teaching Is Problem Solving had the opportunity to get out and work with students in individual settings and classrooms as we were making sense of these place-value ideas.

Wendy Bray: And this isn’t everything. We don’t think that what we’re going to talk about is all of place-value understanding, right? This is what is in the literature as understandings that are well-documented and that are backed up with evidence.

Zak: I think it’s important for us to note that, while we see place value as a much larger thing than just identifying the value of a digit in a certain place, that is a really important understanding for kids to have. We just don’t think that it’s limited to just that one thing. So

we're going to start by taking a look at the first understanding, which is sort of the significance of the position of the digits in a number. Right, like the number, the digit 3 being in the tens place means that there are 3 tens, or there are 30.

Tanya Vik Blais: Okay, I was interviewing a student named Katie, who had just counted a pile of the 35 cubes and written 35 on the paper, and then I asked her some questions about each of the digits in that number.

Tanya Interviews Katie

Tanya: Does this part of the 35 have anything to do with how many cubes you counted?

Katie: Yes.

Tanya: Tell me more about that.

Katie: The 10, because, the three part of the 35 counts as the tens place. So, if I didn't have tens, there would just be 35 ones so I wouldn't have anything else to do with the three.

Zak: This is a little bit more robust of an understanding than just identifying the 3 in the tens place as we've kind of talked about. This is sort of connecting the 3 in the tens place to the sort of set of cubes in front of Katie. So let's take a look at another understanding—understanding 2—in this module of what we see as a strong place value understanding in kindergarten, first-, and second-grade students. That second understanding is sort of grouping-by-tens concept. It's part of our base-10 number system—this idea that 10 ones is the same as 1 ten and 10 tens is the same as 1 hundred and on, and on, and on. We're going to look at a student in this one who's doing a different task.

Wendy: So I showed Valerie this series of task cards that had pictures that kind of resembled base 10 blocks. There were some sticks of ten and some loose ones and then some on the later cards, there were some hundreds. So on this card, Valerie is going to see a representation of 100, a 10 by 10 set of squares, and 2 tens, and 4 loose squares. And, I ask her, “how many small squares are here?” and this is what she did.

Wendy Interviews Valerie

Valerie: 124.

Wendy: Tell me how you figured out 124.

Valerie: Because since there's 10 tens in this, then this is already 100. 100 plus 20—which is these two sticks—is 120. And 120 plus these 4 is 124.

Wendy: So, she didn't immediately know that that 10 by 10 group of squares was 100. She counted 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. She knew that 10 tens was 100. That's 100. And so that's that idea that grouping—she was able to put together those ideas that those 10 tens were 100. And already that those 10 ones were one 10. So, that's great. That's just what we're looking for.

Zak: It does seem like Valerie has a fairly robust understanding at this point, with this task card, of how to, how this organization of 124 can be counted fairly efficiently.

Tanya: So another understanding of place value is for students to be able to notice patterns in our base 10 number system.

Wendy: So, first there's that idea that just having our counting sequence that we use the same 10 digits—1, 2, 3, 4, 5, 6, 7, 8, 9, and 0—to make any number. So that when you're counting and

moving through the counting sequence they just repeat over and over and over in any one place. So, that's a really neat and interesting thing for kids to notice and to start to use to organize their thinking about how numbers get bigger or smaller.

Tanya: I think another really interesting pattern that kids notice is if they're trying to solve a problem like 20 plus 40, they might say—we often hear kids say—2 plus 4 is— 2 plus 4 is 6, so 20 plus 40 is 60, and often they know that because 2 tens plus 4 tens is 6 tens. 6 tens is 60.

Zak: Yeah, there's also this other place value idea that kids engage with when they're learning to add and subtract multiples of ten from a given number, right? If you're subtracting 10 from 34, you notice that the digit in the ones place doesn't change. It's the tens place that changes. If you're adding 30 to a number like 34, the tens place is the only place that changes when you solve that problem.

Tanya: It's also interesting for me, working with third graders that they use these other patterns that they've learned previously about 10 tens is one hundred and they're often really surprised to see that the pattern continues into much larger numbers that 10 hundreds is 1 thousand. That's a big surprise to them. I think in the past, I've always thought that that was understood, that they would generalize that without having to be able to see it. These third graders really—that was a big surprise to them that pattern does continue beyond two-digit numbers or three-digit numbers.

Rob: But maybe that's where place value understanding really picks up and it gets—okay, it's time to get—it's time to really sort of dial this in and teach the conventions of this and teach the base 10 number system.

Zak: This fourth understanding that we're going to explore now is about the idea that numbers can be represented with different but equivalent groupings. This idea that 35 can be seen as 3

tens and 5 ones: that's a really convenient way to think about 35 a lot of times. However, we also know that sometimes kids will want to think about 35 as 2 tens and 15 ones, or other ways to think about the number 35 not just based on the sort of number of tens and numbers of ones in that number.

Rob: But something that's not quite as obvious that is much more complicated for young children to understand who maybe see that 35 can be thought of 3 tens and 5 ones at a glance, that a number like 135 can also be thought of as, not just 1 one-hundred, 3 tens, and 5 ones, but as 13 tens and 5 ones, right? Or, perhaps 1 one-hundred and 35 ones or something like that. But, seeing all of these connections among these numbers is really a nontrivial, another nontrivial leap in understanding of place value.

Zak: This idea of thinking about numbers in these different ways is convenient for students—the flexibility in thinking about 135 as 1 hundred and 2 tens and 15 ones, or some other convenient decomposition. That's important because it allows them to have access to solving computation problems, right? That's one of the reasons we want them to think about these numbers in different groupings. And that leads us right in to the last understanding. Understanding 5 that we'll explore in this module, which is how students use place value knowledge to leverage flexible methods for addition and subtraction. Alright, so we're going to watch a video of—we're going to watch a video of Drew now, a second grader. Drew had 16 cubes in front of him. There was a group of 10 in a cup and then 6 extra ones. We're going to ask Drew that if he were to get 25 more cubes, how many cubes he would have then.

Zak Interviews Drew

Zak: *Let's pretend that I'm going to give you 25 more cubes How many cubes would you have then?*

Drew: *41.*

Zak: *41? How did you get 41?*

Drew: *10 plus 20 equals 30, but then you have 5 plus 6, and 5 plus 5 equals 10 and you have an extra one so then you add 10, carry the 10 into the cup and then you get 41.*

Zak: *Oh, so you were seeing it like making another cup with those extra ten? Okay*

Rob: But one of the things I notice really is how quickly he added 16 and 25 in his mind within 2 to 3 seconds, something like that, and had the right answer. It took him longer to explain what occurred up there, but a lot occurred up there as far as sort of adding the 20 and the 10 and adding the 5 and the 6 part by decomposing the 6 into 5 and 1 and doing the two fives and so on. He also used—I noticed he worked from left to right in place values instead of from right to left in doing this. These conversations are fun because, every time we talk about place value, it just becomes more and more clear to me that these ideas are embedded in our base-10 number system that almost the whole world uses. The Arabic numeral system, with place-value ideas and based on 10, is multifaceted, and for those of us who have had some fair understanding of these ideas for decades, we think—we forget—what it's like to not know that. I think these 5 understandings help us as teachers of children to really develop a better perspective on their view and their understanding of these ideas and it makes it really fun to them. Talk to children, watch what they do, and use what they see and observe and what they show us to understand where they're at in

that journey as they develop a more robust, more integrated understanding of numbers and place value.

Zak: So the next two modules we're going to investigate are framework for how students understand place value and explore classroom applications through viewing video from a classroom working on some of these place value concepts.