

Assessing Early Elementary Students' Place Value Understanding

A Set of Interview Tasks

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Research Report No. 2017-28

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Introduction

This document contains a set of materials we created, selected, or adapted for the purpose of assessing children's understanding of place value. These materials were sequenced to form a diagnostic assessment for use in a one-on-one, interview-type setting. The resulting collection of tasks and questions require approximately 20–30 minutes to administer with an individual child. The assessment is designed for diagnostic purposes. It is not intended to generate an achievement score.

This report provides necessary materials for conducting the interview—including background information about the design of the interview, a materials list, an interview script (or protocol), and blackline masters for printable interview materials.

In our own use of this interview, we aim to keep the focus on diagnostic assessment rather than teaching. Therefore, we avoid providing evaluative feedback on a child's responses and avoid bringing an incorrect answer to a child's awareness. In efforts to maintain a positive, encouraging tone, we are quick to praise effort and explanation of mathematical thinking. To that end, the protocol encourages statements such as, "I appreciate the way you explained that," or "I see just how you were thinking about that."

The interview protocol expects the interviewer to make in-the-moment decisions to modify or omit tasks if the interviewer thinks there is sufficient evidence that a given task will frustrate the child or cause him or her stress. Administered in this manner, we have found that children generally enjoy participating in this interview.

The set of tasks and instructions for interviewers were informed by research-based frameworks for making sense of student thinking related to place value. While they are not discussed in detail in the present report, there are several publications that influenced our thinking. In particular, our thinking was shaped by reports of original research published in refereed journals including Chan, Au, and Tang (2014); Fuson (1990); Fuson, Wearne, et al. (1997); Jones and Thornton (1993); and Ross (1989, 2002). We also gained insights from books that summarize and interpret original research including Carpenter, Fennema, Franke, Levi, and Empson (2015); Richardson (1999); Van de Walle, Lovin, Karp, and Bay-Williams (2018); and Wickett and Burns (2002).

We do not consider this set of interview tasks to comprise a comprehensive assessment of all facets of place value understanding. However, we have found this interview to provide instructionally useful information in a short period of time, and it has supported our own efforts to understand children's thinking related to place value. We hope the readers of this report will find these tasks useful and informative too.

Overview of Place Value Interview Tasks

This interview is organized into three parts that can be completed with a child all at one time (in 20 – 30 minutes) or on three separate occasions. In the text that follows, we will provide a brief overview of the tasks that comprise each of the three parts of the interview. The full set of interview procedures, including specific questions to ask and guidance on what to observe, is offered later in the document. Additionally, the appendices to this document contain an interviewer recording sheet and print materials (e.g., task cards) needed to conduct the interview.

Part One: Counting Cubes, Meaning of Digits, & Mental Addition Task

The first part of the interview engages students in a sequence of four tasks focused on counting, organizing, and discussing sets of cubes. Adapted from Ross (1989, 2002), two parallel tasks prompt children to count a set of cubes, state verbally the number of cubes, write the numeral that matches the set of cubes, and finally, explain the meaning of the digits in the written numeral. The difference between the first two tasks is the way in which the cubes are presented. On the first task (*Thirty-five Cubes*), 35 loose cubes are poured onto the table, and the child is asked, “How many cubes are there?” In contrast, on the *Twenty-six Cubes with Cups* task, the child starts with a pile of 26 cubes and is directed to put 10 cubes in each of two clear cups before being prompted to determine the number of cubes. Thus the second task provides a physical model with a groups-of-ten structure that prompts some children who counted by ones on the first task to think about tens. These contrasting presentations allow a teacher to observe the extent to which the child employs a ‘thinking in tens’ strategy with and without the groups-of-ten structure.

The third and fourth tasks in part one are both adapted from Wickett and Burns (2002). For the third task, *Change to Sixteen*, the child is asked to change the model of 26 cubes built during the second task (two cups of 10 and 6 loose cubes) into 16. This task provides insight into a child’s understanding of the relationship between the quantities 26 and 16. Some children ‘just know’ that 16 is ten less than 26 and quickly remove a cup of 10 cubes. Other children need to count the 16 cubes before discarding the rest. The fourth and final task of this part of the interview prompts the child to mentally compute the sum of 16 and 25. Building on the physical model created by the third task (*Change to 16*), the child is asked to pretend that he or she is given 25 cubes to add to the 16 cubes in front of him. However, some children are not yet comfortable using a mental computation strategy to think about $16 + 25$; so the cubes, paper, and marker used on the previous tasks are intentionally left available for use if requested. The $16 + 25$ task offers insight into whether the child is able to complete the computation mentally and the extent to which his or her strategy involves thinking in tens.

Part Two: Groups of Ten Word Problems

The second part of the interview engages children in solving contextualized multiplication and division word problems involving groups of ten. Such problems are similar to questions typically asked to assess place value understanding but are placed in a real-world context. For example, the interview includes the multiplication-grouping problem, “Your teacher has 4 new boxes of markers. There are 10 markers in each box. How many new markers does she have?” This is a contextualized version of the question, “How much is four tens?” Part two also includes the measurement-division problem, “Your teacher has 30 pretzels. She wants to put the pretzels in snack bags so there are 10 in each. How many snack bags can she make?” This is a contextualized version of the question, “How many tens are in 30?” The strategies children devise for solving contextualized groups of ten problems offer insight into their place value understanding (Carpenter et. al, 2015). For instance, on the problem involving finding the number

of markers in four boxes of ten, some children 'just know' that it is 40. Other children need to use manipulatives or pictures to represent some or all of the elements in the story. Some children can simply represent the boxes and count, "10, 20, 30, 40," while others need to represent 10 markers in each box and count all of the markers by ones. Depending on how a child solves these two tasks, the interviewer has the option to use the boxes of markers or pretzels in a bag context to probe the child's facility with using place value knowledge to solve problems with larger quantities.

Part Three: Strategic Counting Tasks

The third part of the interview is comprised of a sequence of strategic counting tasks adapted from Chan, Au, and Tang (2014). The strategic counting tasks are presented on a series of task cards that show pictures of small squares arranged with more and less obvious base-ten configurations. For example, 10 small squares are sometimes presented in a connected column such that they resemble a base-ten rod. At other times, the small squares are presented in columns of five or two unconnected small squares. For each card, children are asked to determine the number of small squares. Children's ways of counting the small squares on the cards that appear early in the sequence of tasks offer opportunity to observe the circumstances under which the child counts by ones or by tens. For example, some children readily count by tens when ten small squares are configured like a base-ten rod but do not look for opportunities to structure the unconnected 'loose squares' in groups of ten. This task also provides opportunity to observe if a child is able to determine the quantity that is associated with a certain number of tens without counting by tens (e.g., I see four tens. That is 40.) For children who demonstrate facility with the strategic counting tasks within 100, additional task cards provide opportunity to engage the child with larger quantities.

Materials Needed for Place Value Interview

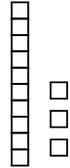
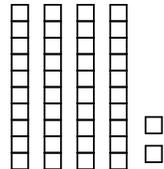
Interview Section	Materials needed
Part One: Counting Cubes, Meaning of Digits, and Mental Addition Task	Bag of 35 loose cubes that are all one color Bag with 26 cubes that are all one color (preferably a different color than the first set) and two clear cups Blank paper Two markers of different colors – One for student, one for interviewer
Part Two: Groups of Ten Word Problems	Word problem cards (See Appendix B) Snap cubes or unifix cubes (some loose and some connected in 'sticks' of 10) Base-ten blocks (unit cubes, rods, flats) Blank paper One marker or pencil
Part Three: Strategic Counting Tasks	Strategic counting task cards (See Appendix C)
General	Interviewer recording sheet (See Appendix A) Pen or pencil for interviewer

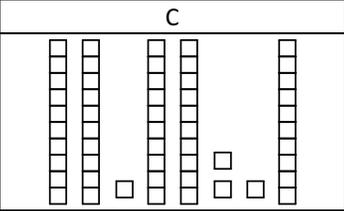
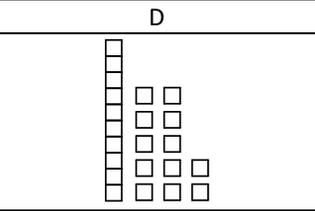
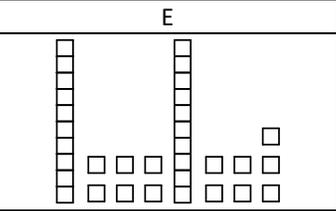
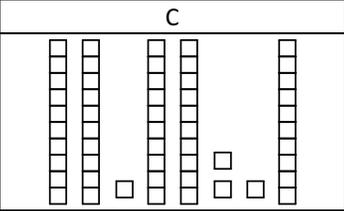
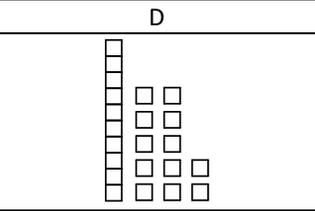
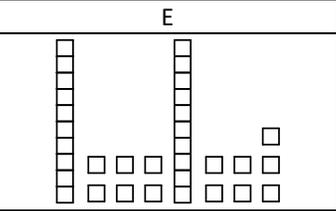
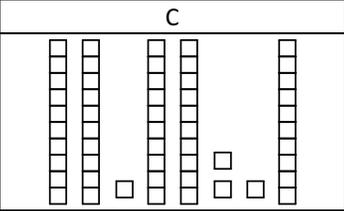
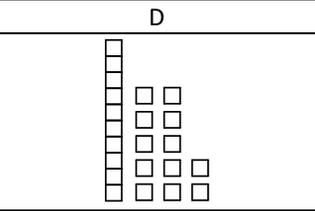
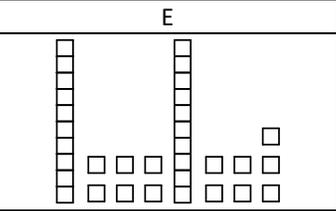
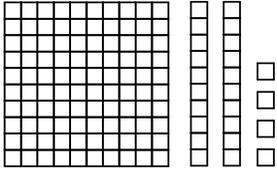
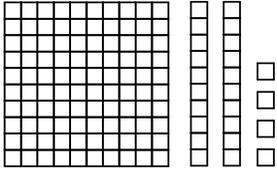
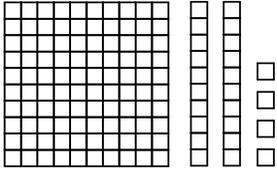
Procedures for Place Value Interview

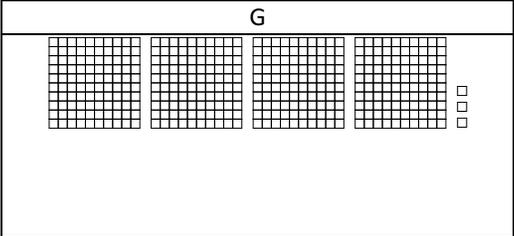
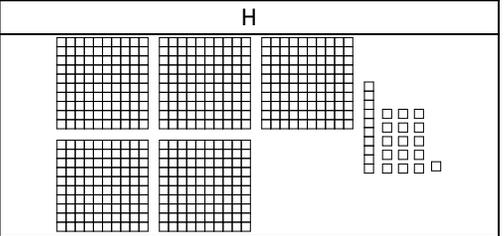
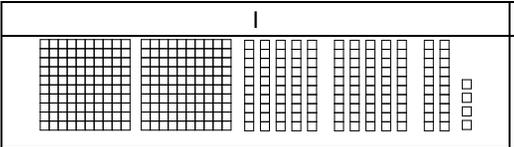
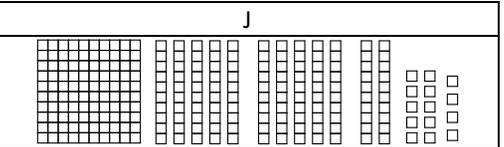
PART ONE: COUNTING CUBES, MEANING OF DIGITS, & MENTAL ADDITION TASK		
Task	Interview Procedure	What to Observe
<p>1A <i>Thirty-five Cubes</i></p> <p><i>(adapted from Ross, 1989, 2002)</i></p>	<ol style="list-style-type: none"> 1. Pour 35 loose cubes on the table in front of the child. 2. Ask: How many cubes are there? <ol style="list-style-type: none"> a. (Optional) If child reports an incorrect count, consider prompting a recount. 3. After counting, give the child paper and a marker. Ask: Can you write down that number? 4. Circle the number in the ones place and ask: Does this part of the [number written] have anything to do with how many cubes you have? <ol style="list-style-type: none"> a. If the child says “yes,” ask: Can you tell me more about that? b. If the child says, “no,” just move on. 5. Circle the number in the tens place and ask: Does this part of the [number written] have anything to do with how many cubes you have? <ol style="list-style-type: none"> a. If the child says “yes,” ask: Can you tell me more about that? b. If the child says, “no,” just move on. 6. Move the 35 cubes from this task to the side in preparation for the next task. 	<p>Tasks 1A and 1B</p> <ul style="list-style-type: none"> • Does the child organize the loose cubes in the <i>Thirty-five Cubes</i> task into tens or any other kind of grouping to facilitate ease of counting and accuracy? • Does the child’s way of determining how many cubes use ‘thinking in tens,’ or does the child consistently seem to be thinking in ones? • If there is evidence of ‘thinking in tens,’ does the child appear to ‘just know’ that three tens is 30, or does the child need to count 10, 20, 30? • When asked about the relationship between the digits in each written numeral (i.e., 35 and 26) and the cubes, is the child able to explain what each digit represents on both tasks, on the <i>Twenty-six Cubes in Cups</i> task only, or on neither task?
<p>1B <i>Twenty-six cubes with cups</i></p> <p><i>(adapted from Ross, 1989, 2002)</i></p>	<ol style="list-style-type: none"> 7. Pour out the 26 loose cubes, and place the 2 clear cups on the table. 8. Say: Can you put 10 cubes in this cup? (When finished) Can you put 10 cubes in this cup? Leave the 6 extra cubes loose on the table. 9. Say: Each cup has 10 cubes in it. (Gesture to the cups.) And we have some extra cubes. (Gesture to the extra cubes.) How many cubes is that altogether? <ol style="list-style-type: none"> a. (Optional) Can you tell me how you got that? b. (Optional) If the child reports an incorrect count, consider prompting a recount. 10. After counting, give the child paper and a marker: Can you write down that number? 11. Circle the digit in the ones place and ask: Does this part of the [number written] have anything to do with how many cubes are here? <ol style="list-style-type: none"> a. If the child says “yes,” ask: Can you tell me more about that? b. If the child says, “no,” just move on. 12. Circle the digit in the tens place and ask: Does this part of the [number written] have anything to do with how many cubes are here? <ol style="list-style-type: none"> a. If the child says “yes,” ask: Can you tell me more about that? b. If the child says, “no,” just move on. 	

PART ONE CONTINUED: COUNTING CUBES, MEANING OF DIGITS, & MENTAL ADDITION TASK		
Task	Interview Procedure	What to Observe
<p>1C <i>Change to 16</i></p> <p><i>(adapted from Wickett & Burns, 2002)</i></p>	<p>13. Say: You just counted 26 cubes. Can you change it into 16 cubes?</p> <p>a. If the child only responds with a yes, ask: Can you show me how you would do that?</p> <p>b. If the child does not have 16, work with the child to “check” that it is 16 and make changes – so that the next task starts with 16. It is okay to count the 16 with the child after the child has attempted the task.</p> <p>14. In preparation for the next task, push the excess cubes (those not included in the set of 16) to the side but still visible on the table.</p>	<ul style="list-style-type: none"> How does the child change the cubes to 16? Does the child... <ul style="list-style-type: none"> Immediately remove one cup of 10 cubes, suggesting knowledge that 16 is ten less than 26? Locate the 16 using ‘thinking in tens’ (this is 10 and this is six) before removing the remaining 10? Count the 16 by ones, before removing the remaining 10? Dump out all 26 cubes, then count and remove until there are 16?
<p>1D $16 + 25$</p> <p><i>(adapted from Wickett & Burns, 2002)</i></p>	<p>15. Say: Now you have 16 cubes. Let’s pretend that I give you 25 more cubes. How many will you have then?</p> <p>***If the child asks for tools (cubes, cups, and/or paper and marker), allow him or her to use them.</p>	<ul style="list-style-type: none"> How does the child use knowledge of tens and ones to solve? Does the child... <ul style="list-style-type: none"> Ask for cubes or pencil and paper to solve? Start at 16 and count on by ones for 25 counts (suggesting thinking in ones)? Start at 16 and add 25 in increments of ten (e.g., $16 + 10 = 26$, $26 + 10 = 36$, $36 + 5 = 41$)? Combine like units (e.g., $10 + 20 = 30$, $6 + 5 = 11$, $30 + 11 = 41$)?

PART TWO: GROUPS OF TEN WORD PROBLEMS		
Task	Interview Procedure	What to Observe
<p>2A <i>Groups of Ten Word Problem, Multiplication Grouping</i></p>	<p>1. Say: For the next part, I am going to ask you to solve some story problems. If you want, you can solve them mentally, with just your brain, or you can use your fingers or any of these tools to help you. [Present paper, markers, base ten blocks, and snap cubes and check for understanding of each tool.] I am going to read each story problem aloud. If you want me to read it again, you just have to ask. Are you ready for the first one?</p> <p>2. Present the child with the following multiplication grouping problem on a card and read the problem aloud. Ask follow-up questions, as needed, to understand the child's strategy for determining an answer.</p> <p><i>Your teacher has 4 new boxes of markers. There are 10 markers in each box. How many new markers does she have?</i></p> <p>3. (OPTIONAL) If the child answers the first question very quickly or with a 'thinking in tens' strategy, consider posing one or more additional follow-up problems that utilize the same context but involve larger quantities.</p> <p><i>Your teacher has ____ new boxes of markers. There are 10 markers in each box. How many new markers does she have?</i> Number options: 13, 27</p>	<ul style="list-style-type: none"> If the child uses manipulatives or pictures to represent each group of ten in accordance with the problem, does the child use a strategy that reflects thinking in ones or thinking in tens? <ul style="list-style-type: none"> - Is each group of ten represented as ten ones or one ten? - Does the child count the individual objects in each group by ones? - Does the child use counting by tens (10, 20, 30, 40)? - Does the child recognize the relationship between the number of units of ten and the total without counting (e.g., 4 tens is 40)? - To what extent is the child able to think flexibly about the representation of a set of ten as simultaneously being one ten and ten ones (e.g., both one box and ten markers)?
<p>2B <i>Groups of Ten Word Problem, Measurement Division</i></p>	<p>4. Present the child with the following measurement division problem on a card and read the problem aloud. Ask follow-up questions, as needed, to understand the child's strategy for determining an answer.</p> <p><i>Your teacher has 30 pretzels. She wants to put the pretzels in snack bags so there are 10 in each. How many snack bags can she make?</i></p> <p>5. (OPTIONAL) If the child answers the first question very quickly or with a 'thinking in tens' strategy, consider posing one or more additional follow-up problems that utilize the same context but involve larger quantities.</p> <p><i>Your teacher has ____ pretzels. She wants to put the pretzels in snack bags so there are 10 in each. How many snack bags can she make?</i> Number options: 70, 120, 240</p>	<ul style="list-style-type: none"> If the child does not use manipulatives or pictures to represent each group of ten, how does the child solve the problem? <ul style="list-style-type: none"> - Does the child use a strategy that requires time to figure out the answer, such as counting by tens, repeated addition, or repeated subtraction? - Does the child 'just know' the answer quickly?

PART THREE: STRATEGIC COUNTING TASKS		
<p>General Procedure: Present the strategic counting tasks (adapted from Chan, Au, & Tang, 2014) to the child one-at-a-time on individual cards. If the child counts by ones for the first two tasks, the interview can be terminated because it can be assumed that the child would continue to count by ones on the more difficult cards. If the child uses some base ten structuring in his/her count, the interviewer should continue with the additional items. However, if the child appears to hit his or her ceiling and/or is consistently counting in an inefficient way, the interview should be ended.</p>		
Task	Interview Procedure	What to Observe
3A	<p>1. Use task card A to support the child's understanding of how this set of tasks is going to work. Say: Next, I am going to show you some cards with pictures of small squares. Let's look at the first one. (Display Card A.)</p>  <p>2. Say: Some of squares are touching and some are not. Can you point to two squares that are touching? (If the child does not readily identify two square that are not touching, help the child to locate them.) How many small squares are touching here (gesture to the 'stick' of 10 small squares)? (If needed, help the child determine that the 'stick' contains 10 small squares.) Next I want you to tell me how many small squares are on the card.</p>	<ul style="list-style-type: none"> Does the child understand the task? <ul style="list-style-type: none"> Does the child understand what constitutes a 'small square'? Does the child understand to count the squares that are touching AND those that are not? Does the child appear to recognize the 'stick' of connecting squares as a representation of 10 without counting? If they do, is it before or after they count the cubes on the stick?
3B	<p>3. Present task card B. Say: How many small squares are on this card?</p>  <p>4. If the child's way of determining the number of squares is unclear, ask questions to clarify (e.g., How did you figure out 42?)</p>	<ul style="list-style-type: none"> Does the child count the small squares in each 'stick' of 10 by ones, or does the child use knowledge that the stick is 10? If 'thinking in tens,' does the child... <ul style="list-style-type: none"> Count by tens to 40? Recognize that there are four tens and 'know' that is 40 (without counting by tens)?

PART THREE CONTINUED: STRATEGIC COUNTING TASKS								
Task	Interview Procedure	What to Observe						
	<p>5. Terminate the interview here (after task 3B) if the child has exclusively used 'counting by ones' on the first two tasks or if the child has responded by guessing. Say: That was the last problem. Thank you so much for sharing your math thinking with me today. Otherwise, proceed to the next task card.</p>							
3C – 3E	<p>6. Present task cards C, D, and E, one at a time. When presenting each new task card, say: How many small squares are on this card?</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th>C</th> <th>D</th> <th>E</th> </tr> </thead> <tbody> <tr> <td style="text-align: center;">  </td> <td style="text-align: center;">  </td> <td style="text-align: center;">  </td> </tr> </tbody> </table> <p>7. If the child's way of determining the number of squares is unclear, ask questions to clarify.</p>	C	D	E				<ul style="list-style-type: none"> Does the child always count the small squares from left to right, regardless of the configuration? Does the child consistently treat connected 'sticks' of ten as 'tens,' or are the sticks sometimes counted by ones? When the child recognizes 'tens,' does the child count by tens, or does the child recognize units of ten and determine the quantity without counting by tens (e.g., There are 5 tens, so 50.) Does the child group a set of ten 'loose' ones into one ten?
C	D	E						
								
3F	<p>8. Present task card F. Say: How many small squares are on this card?</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tbody> <tr> <td style="text-align: center;">  </td> </tr> </tbody> </table> <p>9. If the child's way of determining the number of squares is unclear, ask questions to clarify.</p> <p>10. Decide whether to terminate or continue the interview based on the child's response. If terminating the interview, say: That was the last problem. Thank you so much for sharing your math thinking with me today.</p>		<ul style="list-style-type: none"> How does the child determine the quantity of the 10x10 grid of small squares (e.g., guessing, counting by ones or tens, recognizing that it is 100 without counting)? How does the child handle the transition crossing 100? <ul style="list-style-type: none"> Does the child associate the appropriate value with each type of unit (i.e., small squares are treated as ones and 'sticks' are treated as tens)? Are the 'sticks' of ten counted by ones (i.e., 100, 101 102...120), counted by tens (i.e., 100, 110, 120), or does the child recognize the two units of ten as 20 and then combine with 100? 					
								

PART THREE CONTINUED: STRATEGIC COUNTING TASKS		
Task	Interview Procedure	What to Observe
3G – 3J	<p>11. Say: On this next card, I still want you to tell me how many squares. But the squares are smaller.</p> <p>12. Present task cards G, H, I, and J, one at a time. When presenting each new task card, say: How many small squares are on this card?</p> <div style="display: flex; justify-content: space-around; border: 1px solid black; padding: 5px;"> <div style="border: 1px solid black; padding: 5px; width: 45%;"> <p style="text-align: center;">G</p>  </div> <div style="border: 1px solid black; padding: 5px; width: 45%;"> <p style="text-align: center;">H</p>  </div> </div> <div style="display: flex; justify-content: space-around; border: 1px solid black; padding: 5px; margin-top: 10px;"> <div style="border: 1px solid black; padding: 5px; width: 45%;"> <p style="text-align: center;">I</p>  </div> <div style="border: 1px solid black; padding: 5px; width: 45%;"> <p style="text-align: center;">J</p>  </div> </div> <p>13. If the child's way of determining the number of squares is unclear, ask questions to clarify.</p> <p>14. Terminate the interview. Say: That was the last problem. Thank you so much for sharing your math thinking with me today.</p>	<ul style="list-style-type: none"> On tasks G & H: How does the child determine the quantity of the hundreds? <ul style="list-style-type: none"> - Counting by hundreds - Recognizing or counting the number of units of one-hundred (e.g., There are four hundreds, so that is 400.) - Counting by tens On tasks H & J: Does the child recognize that ten of the 'loose' small squares can be treated as 10, or are these squares counted by ones? On tasks I & J: Does the child recognize that ten 'sticks' of ten can be treated as one unit of 100? If so, what language does the child use to describe ten tens?

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Appendix A: Place Value Interview Interviewer Notes Sheet

Student Name _____

Date _____

Part One: Counting Cubes, Meaning of Digits, & Mental Addition Task

A. *Thirty-five Cubes* (adapted from Ross, 1989, 2002)

Have ready: (1) Bag of 35 loose cubes that are all one color, (2) Blank sheet of paper, (3) 2 Different Color Markers – 1 for student, 1 for interviewer

Pour out 35 loose cubes.

Ask: **How many cubes are there?**

(Optional) If child arrives at an incorrect count, it is an option to encourage a recount.

Number reported verbally: _____

Strategy used to count:

After counting, give the child paper and a marker: **Can you write down that number?**

Number written by child: _____

Circle the number in the ones place and ask: **Does this part of the [number written] have anything to do with how many cubes you have?**

- If the child says “yes,” ask: **Can you tell me more about that?**
- If the child says, “no,” just move on.

Response:

Circle the number in the tens place and ask: **Does this part of the [number written] have anything to do with how many cubes you have?**

- If the child says “yes,” ask: **Can you tell me more about that?**
- If the child says, “no,” just move on.

Response:

At end of task 1, push this set of cubes to the side but leave on table.

B. *Twenty-six Cubes with Cups* (adapted from Ross, 1989, 2002)

Have ready: Bag with 26 cubes (all one color), 2 clear cups, paper, 2 markers

Pour out the 26 cubes, and place the 2 cups on the table.

Say: **Can you put 10 cubes in this cup?** (When finished) **Can you put 10 cubes in this cup?** Leave the 6 extra cubes loose on the table.

Say: **Each cup has 10 cubes in it.** (Gesture to the cups.) **And we have some extra cubes.** (Gesture to the extra cubes.) **How many cubes is that altogether?** (Gesture to the whole set.)

- (Optional) Can you tell me how you got that?
- (Optional) If the child arrives at an incorrect count, it is an option to encourage a recount.

Number reported verbally: _____

Strategy used to count:

After counting, give the child paper and a marker: **Can you write down that number?**

Number written by child: _____

Circle the digit in the ones place and ask: **Does this part of the [number written] have anything to do with how many cubes are here?**

- If the child says "yes," ask: **Can you tell me more about that?**
- If the child says, "no," just move on.

Response:

Circle the digit in the tens place and ask: **Does this part of the [number written] have anything to do with how many cubes are here?**

- If the child says "yes," ask: **Can you tell me more about that?**
- If the child says, "no," just move on.

Response:

C. *Change to 16* (adapted from Wickett & Burns, 2002)

Ask: **You just counted [number reported] cubes. Can you change it into 16 cubes?** Record how the child changes 26 to 16.

- If the child only responds with a yes, ask: **Can you show me how you would do that?**
- If the child does not have 16, work with the child to “check” that it is 16 and make changes – so that the next task starts with 16. It is okay to count the 16 with the child after the child has attempted the task.
- Push the excess cubes (those not included in the set of 16) aside.

Notes on what has occurred (i.e., the child’s strategy, interviewer intervention):

D. *16 + 25* (adapted from Wickett & Burns, 2002)

Say: **Now you have 16 cubes. Let’s pretend that I give you 25 more cubes. How many will cubes you have then?**

- Look to see if the child is able to use knowledge of tens and ones to solve. If they ask for tools (base ten block, cubes, and/or cups), allow them to use them.

Notes on response:

Reset the two bags of cubes for Part One. Elicit the child’s help in getting the cubes back in the bags. Set those bags aside.

Part Two: Groups of Ten Word Problems

Have ready: (1) Word problem cards, (2) Snap cubes or unifix cubes, including some loose and some sticks of ten, (3) Base-ten blocks, including unit cubes, rods, and flats, (4) Paper/pencil

Say: **For the next part, I am going to ask you to solve some story problems. If you want, you can solve them mentally, with just your brain, or you can use your fingers or any of these tools to help you.** [present the paper, markers, base ten blocks, and snap cubes.] **I am going to read each story problem aloud. If you want me to read it again, you just have to ask. Are you ready for the first one?**

The interviewer will then present the child with each problem on a card and will read the problem aloud.

Problem A: Your teacher has ____ new boxes of markers. There are 10 markers in each box. How many new markers does she have?

- Number options: 4, 13, 21
- Use 4 as an opening problem with all children.
- Follow-up with 13 and/or 21 if the child is able to answer fairly quickly (e.g., What if your teacher had 13 boxes of new markers? How many markers would that be?)
- Observe and record the child's strategy for solving. If not clear, ask probing questions (e.g., How did you get X?)

Notes on how the child solved:

Your teacher has 4 new boxes of markers. There are 10 markers in each box. How many new markers does she have?

Your teacher has ____ new boxes of markers. There are 10 markers in each box. How many new markers does she have?

Your teacher has ____ new boxes of markers. There are 10 markers in each box. How many new markers does she have?

Problem B: Your teacher has _____ pretzels. She wants to put the pretzels in snack bags so there are 10 in each. How many snack bags can she make?

- Number options: 30, 70, 120, 240
- Use 30 for the first presentation of this problem.
- Follow-up with additional numbers if the child is able to answer fairly quickly (e.g., What if your teacher had 120 pretzels? How many snack bags of 10 could she make then?)
- Observe and record the child's strategy for solving. If not clear, ask probing questions (e.g., How did you get X ?)

Notes on how the child solved:

Your teacher has 30 pretzels. She wants to put the pretzels in snack bags so there are 10 in each. How many snack bags can she make?

Your teacher has _____ pretzels. She wants to put the pretzels in snack bags so there are 10 in each. How many snack bags can she make?

Your teacher has _____ pretzels. She wants to put the pretzels in snack bags so there are 10 in each. How many snack bags can she make?

Your teacher has _____ pretzels. She wants to put the pretzels in snack bags so there are 10 in each. How many snack bags can she make?

Part Three: Strategic Counting Tasks

Have ready: Strategic counting task cards

The strategic counting tasks (adapted from Chan, Au, & Tang, 2014) that follow will be presented to the child one-at-a-time on individual cards. If a child counts by ones for the first two tasks, terminate the interview. If the child uses some base ten structuring in his/her count, the interviewer should continue with the additional items. However, if the child appears to hit his or her ceiling and/or are consistently counting in an inefficient way, the interview should be terminated.

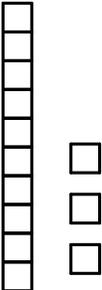
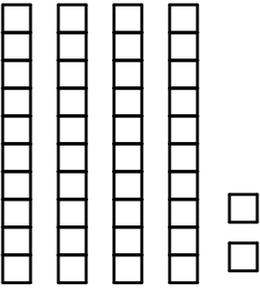
Say: **Next, I am going to show you some cards with pictures of small squares. Let's look at the first one. (Display Card A.) Some of squares are touching and some are not. Can you point to two squares that are touching?**

How many small squares are touching here (gesture to the 'stick' of 10)?

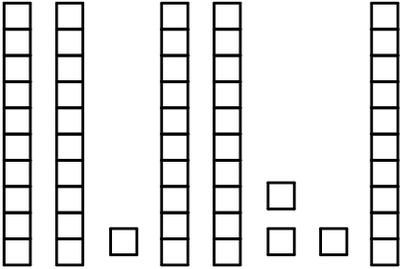
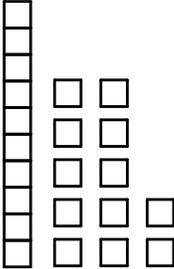
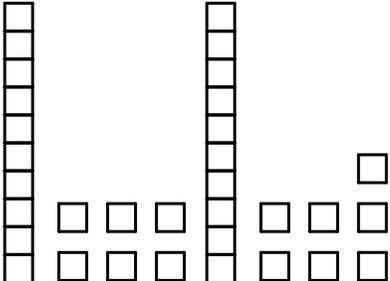
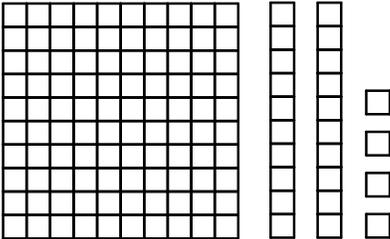
Next I want you to tell me how many small squares are on the card. Clarify that you mean the whole set, including the squares that are touching and those that are not.

Use Card A to make sure the child understands the task.

For the remainder of tasks, present the task card and ask: **How many small squares are on this card?** As needed, ask: **How did you figure out [number stated]?**

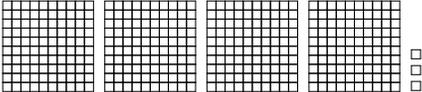
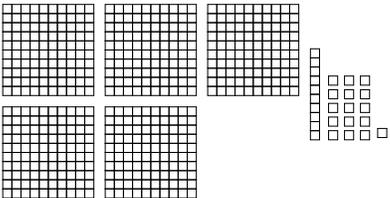
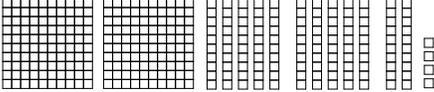
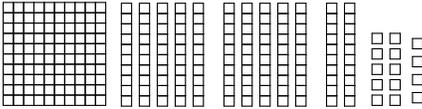
Item	Image	Interviewer Notes on Child's Strategy
A		
B		

If terminating interview, say: **That was the last problem. Thank you so much for sharing your math thinking with me today.** If continuing interview, go to next card.

Item	Image	Interview Notes on Child's Strategy
C		
D		
E		
F		

If terminating interview, say: **That was the last problem. Thank you so much for sharing your math thinking with me today.**

If continuing, say: **On this next card, I still want you to tell me how many squares. But the squares are smaller.** Then present the next card.

Item	Image	Interview Notes on Child's Strategy
G		
H		
I		
J		

Say: **That was the last problem. Thank you so much for sharing your math thinking with me today.**

Appendix B: Groups of Ten Word Problem Cards

Your teacher has 4 new boxes of markers. There are 10 markers in each box. How many new markers does she have?

Your teacher has 13 new boxes of markers. There are 10 markers in each box. How many new markers does she have?

Your teacher has 21 new boxes of markers. There are 10 markers in each box. How many new markers does she have?

Your teacher has 30 pretzels. She wants to put the pretzels in snack bags so there are 10 in each. How many snack bags can she make?

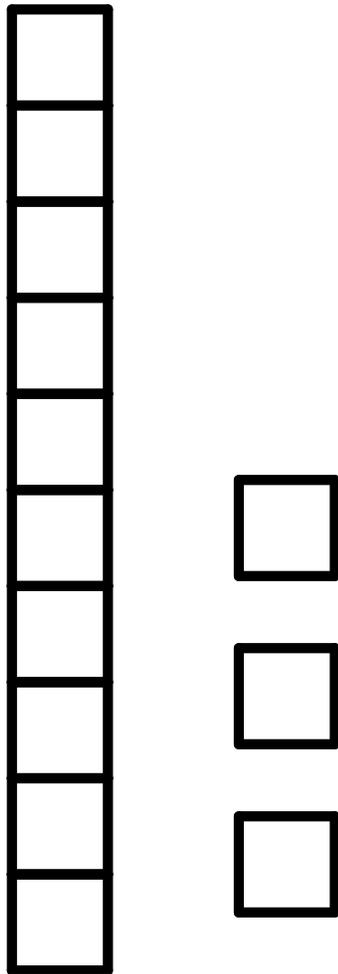
Your teacher has 70 pretzels. She wants to put the pretzels in snack bags so there are 10 in each. How many snack bags can she make?

Your teacher has 120 pretzels. She wants to put the pretzels in snack bags so there are 10 in each. How many snack bags can she make?

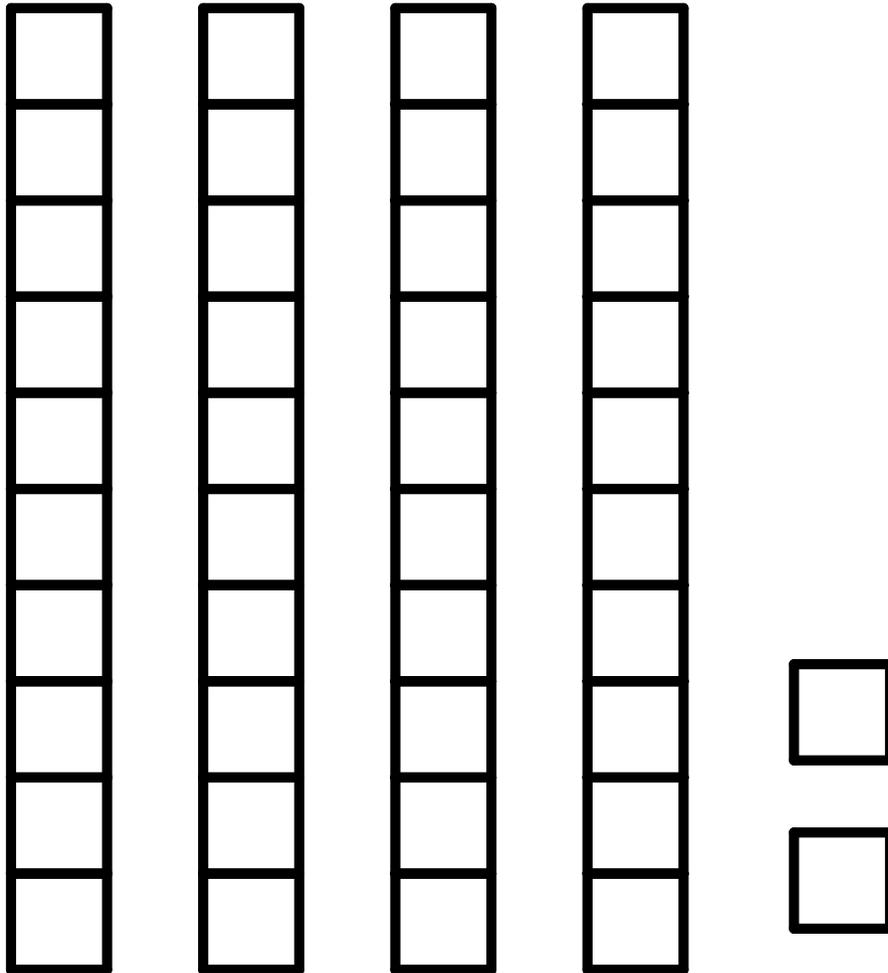
Your teacher has 240 pretzels. She wants to put the pretzels in snack bags so there are 10 in each. How many snack bags can she make?

Appendix C: Strategic Counting Tasks Image Cards

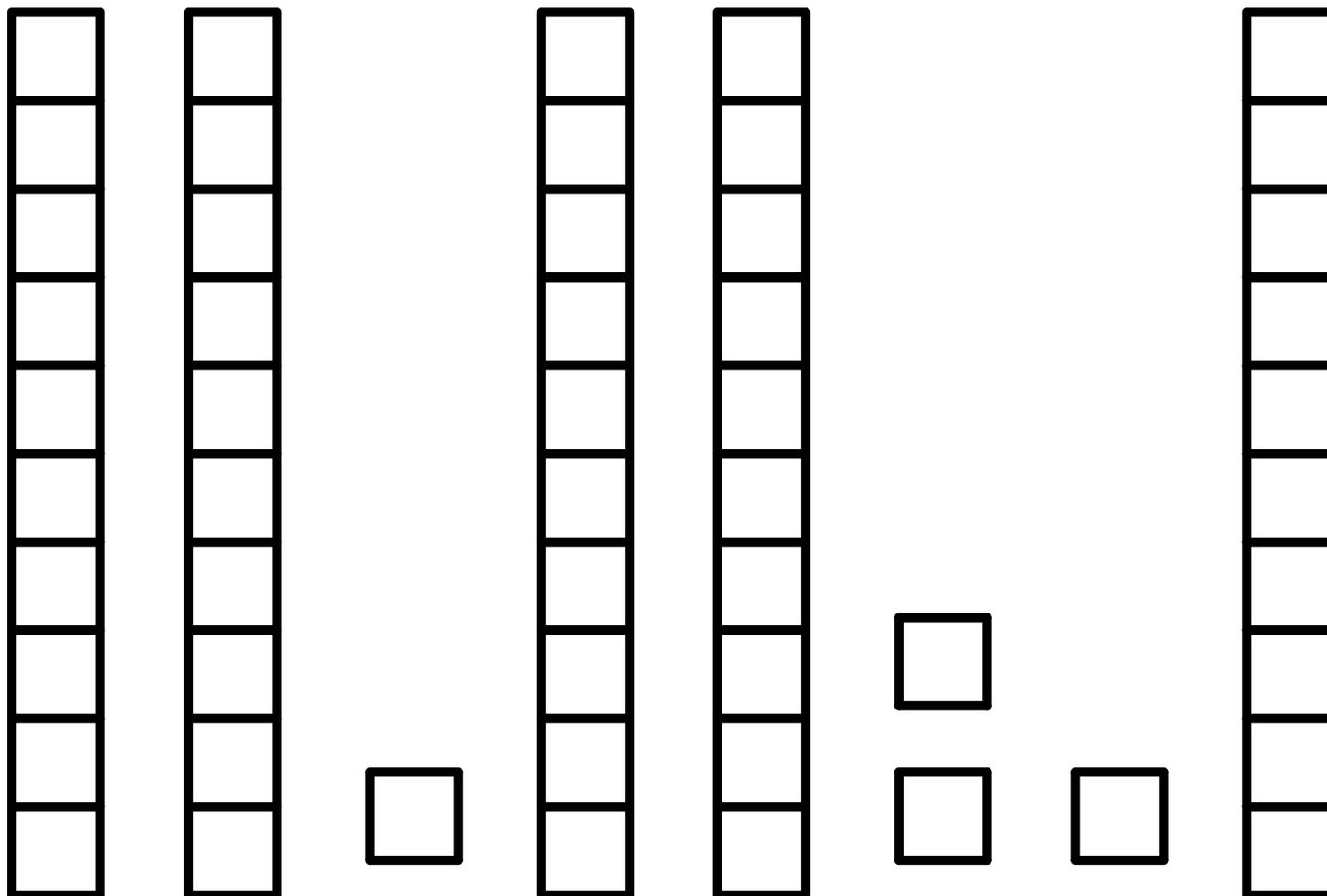
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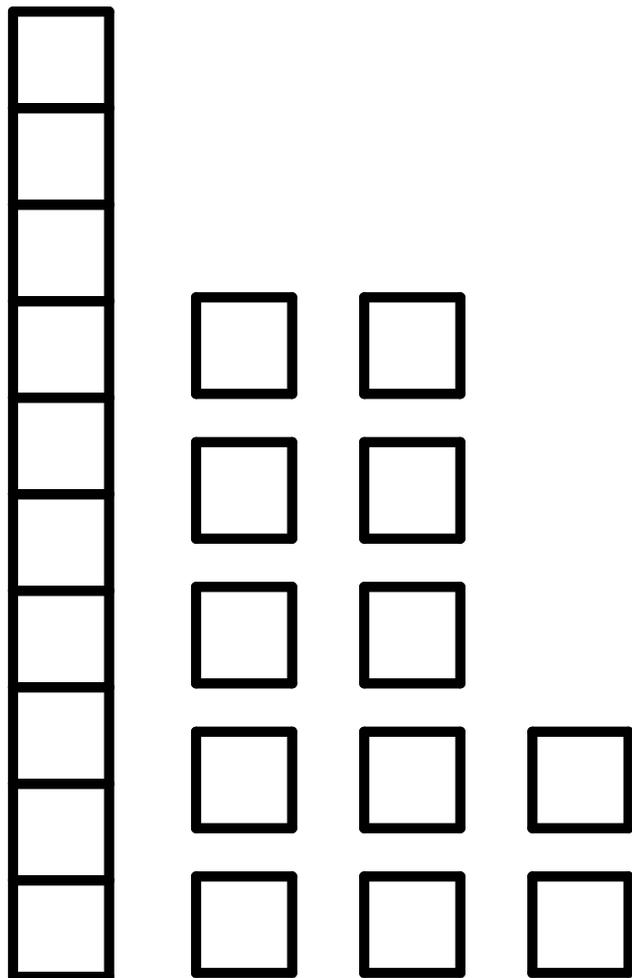
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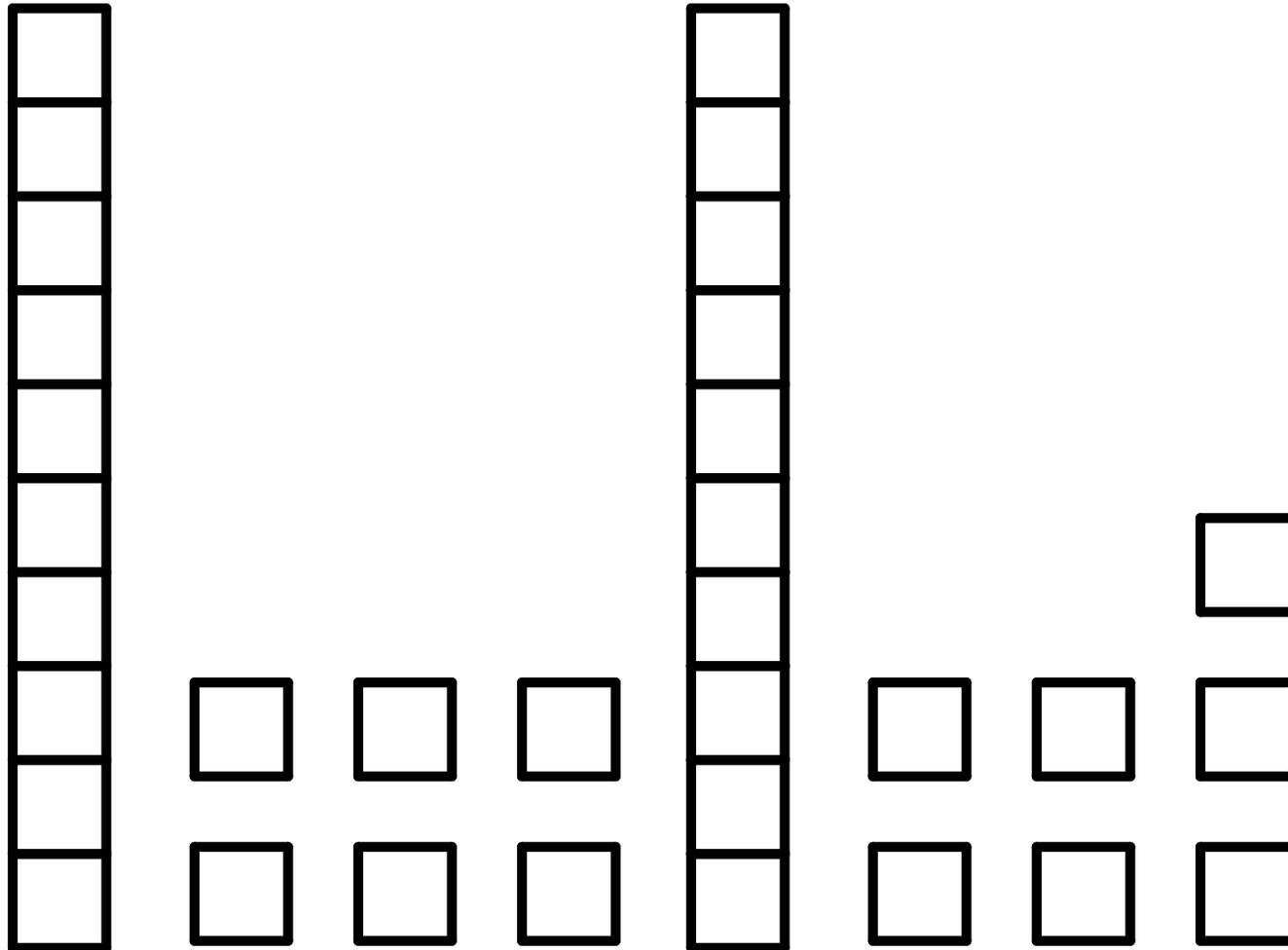
C



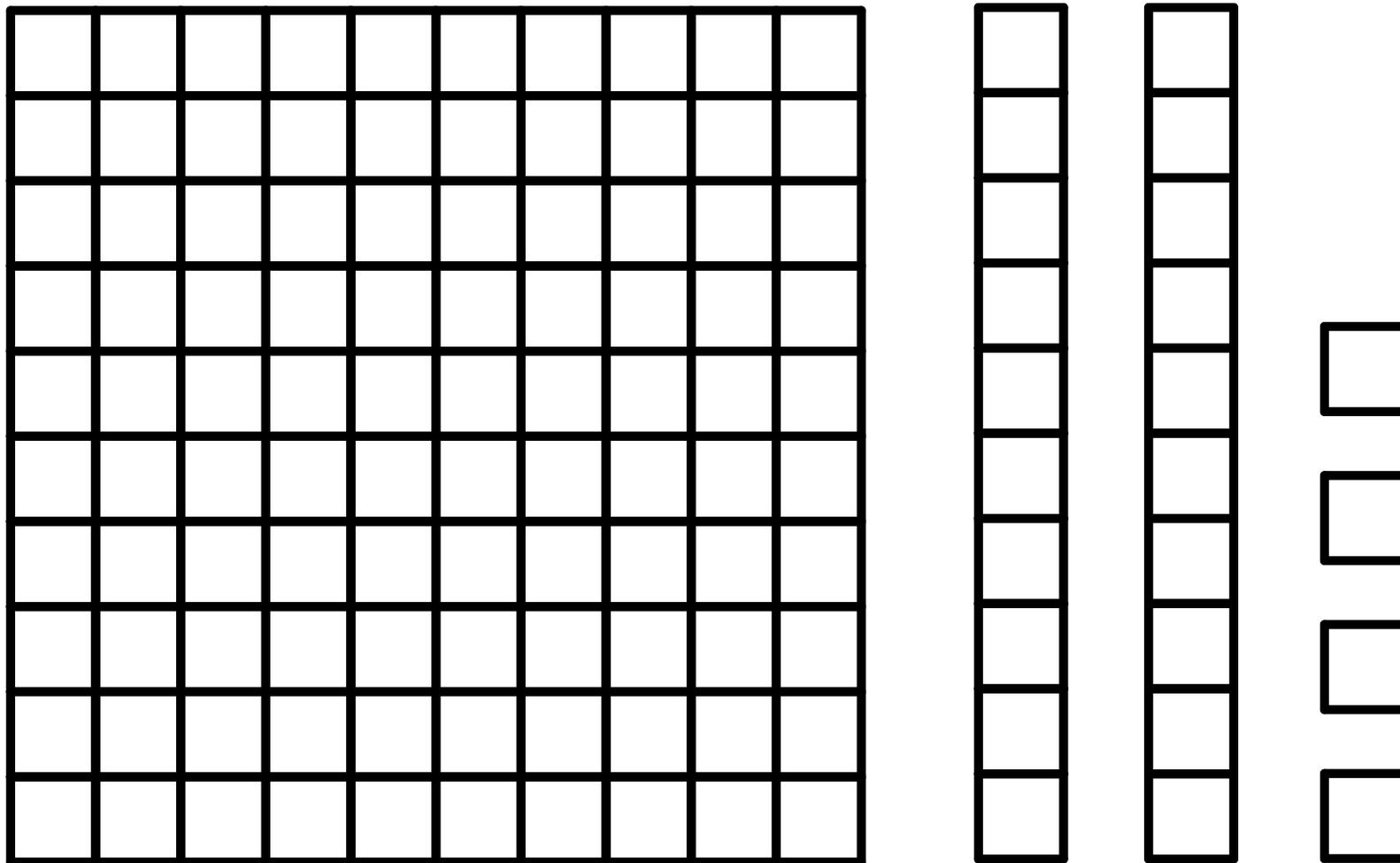
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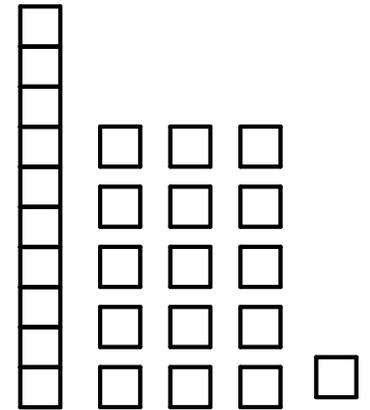
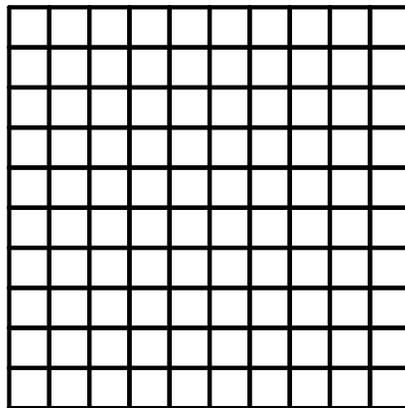
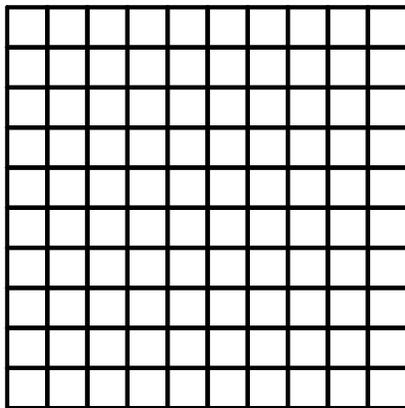
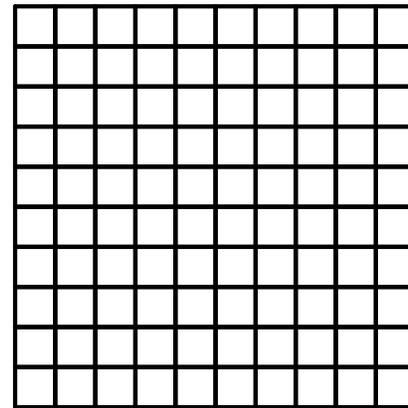
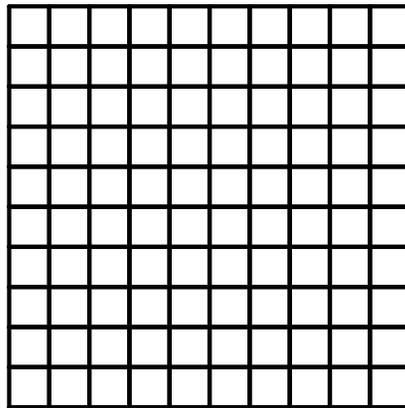
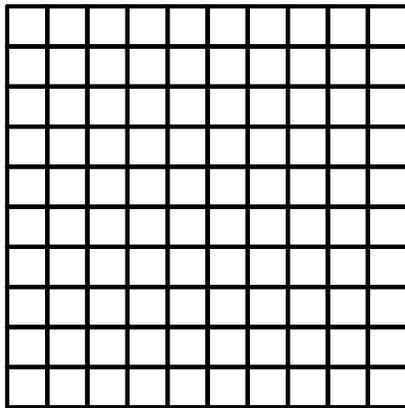
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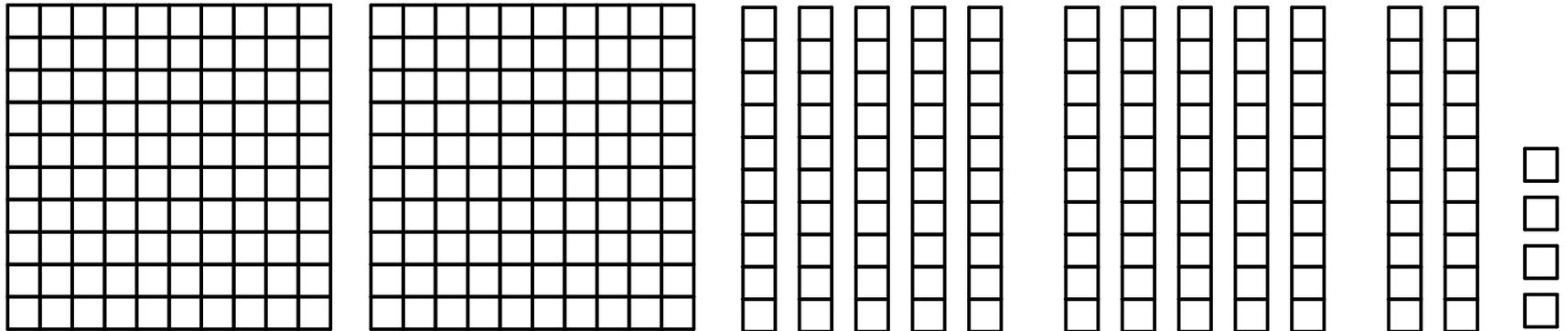
F



H



I



J

